

AlterBBN v2: A public code for calculating Big-Bang nucleosynthesis constraints in alternative cosmologies

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URL: <https://alterbbn.hepforge.org/>

Abstract

We present the version 2 of `AlterBBN`, an open public code for the calculation of the abundance of the elements from Big-Bang nucleosynthesis. It does not rely on any closed external library or program, aims at being user-friendly and allowing easy modifications, and provides a fast and reliable calculation of the Big-Bang nucleosynthesis constraints in the standard and alternative cosmologies.

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1 Introduction

`AlterBBN` is a public C program for the calculation of the abundance of the elements generated during Big-Bang nucleosynthesis (BBN), released under the GPL version 3 license. The first version was released in 2011 [1] and could be considered as a spiritual successor of `NUC123` [2]. Contrary to other public BBN codes such as the Fortran program `ParthENoPE` [3, 4] or the Mathematica code `PRIMAT` [5], the main purpose of `AlterBBN` is to compute the abundance of the elements in alternative cosmological scenarios, in a fast and reliable way. `AlterBBN` is also included in the `SuperIso Relic` package [6–8].

`AlterBBN` can be downloaded from its new website:

<https://alterbbn.hepforge.org/>

and involves an enlarged development team.

In `AlterBBN v2`, automatic calculation of errors and correlations has been implemented using methods similar to the ones described in Refs. [9, 10]. To improve the speed of the calculations, parallel processing is possible through the `OpenMP` library. In addition, the units throughout the code have been unified to have GeV as the main unit, in order to stay consistent with `SuperIso Relic`. The nuclear reaction network has also been extended, and the code has been scrutinized and improved for precision and speed.

Moreover, new cosmological scenarios have been implemented in `AlterBBN`, such as reheating, decaying primordial scalar field, and simple WIMP scenarios.

The rest of this paper is organised as follows. Section 2 provides a review of BBN physics and the cosmological modifications implemented in `AlterBBN`. Section 3 describes the content of the `AlterBBN` package. Section 4 gives usage instructions. Section 5 describes the input and output of `AlterBBN`. Section 6 provides examples of analyses which can be performed with `AlterBBN`. Short descriptions of the nuclear reaction network, integration methods and BBN constraints implemented in `AlterBBN` are given in the appendices.

2 BBN physics and cosmology

In this section we briefly present the physics relevant for BBN studies. We consider the system of natural units $c = \hbar = k_B = 1$.

2.1 Cosmological standard model and BBN

2.1.1 General equations

In the beginning of the BBN epoch, the universe contained photons γ , electrons, e^- , positrons, e^+ , protons p , neutrons n , neutrinos ν and antineutrinos $\bar{\nu}$, and presumably, dark matter χ . During this epoch, new nuclei formed via nuclear reactions

(see Table A.1 in Appendix A), which may be classified under the general name of baryons, b. BBN takes place in the radiation-dominated era of the early universe expansion and depends on the expansion rate, \dot{a} , given by the Friedmann equation as a function of the total density ρ_{tot} ,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\text{tot}}, \quad (2.1)$$

where H is the Hubble parameter and G is Newton's gravitational constant. The total density ρ_{tot} is given by the sum over the previously mentioned densities,

$$\rho_{\text{tot}} = \rho_{\gamma} + \rho_{\nu,\bar{\nu}} + \rho_{\text{b}} + \rho_{e^-} + \rho_{e^+} + \rho_{\chi}, \quad (2.2)$$

where $\rho_{\nu,\bar{\nu}}$ is the energy density of neutrinos and antineutrinos. Each density is governed by continuity equations,

$$\frac{d}{dt}(\rho_{\text{set}}a^3) + P_{\text{set}}\frac{d}{dt}(a^3) - T\frac{d}{dt}(s_{\text{set}}a^3) = 0. \quad (2.3)$$

In general, both ρ_{set} and s_{set} are functions of both temperature, T , and scale factor, a , or – since the scale factor is only time-dependent – as explicit functions of both temperature T and time t . Using the fact that

$$\frac{d\ln(a^3)}{dt} = 3H, \quad (2.4)$$

the continuity equation can be expressed as a relation between the scale factor and temperature,

$$\frac{d\ln(a^3)}{dt} = -3H \frac{\frac{\partial\rho_{\text{set}}}{\partial T} - T\frac{\partial s_{\text{set}}}{\partial T}}{\frac{\partial\rho_{\text{set}}}{\partial t} + 3H(\rho_{\text{set}} + P_{\text{set}}) - T\left(\frac{\partial s_{\text{set}}}{\partial t} + 3Hs_{\text{set}}\right)}. \quad (2.5)$$

The cosmological components are generally considered to be ideal gases, with densities depending only on the temperature, and expansion is assumed to be adiabatic so that $d(sa^3)/dt = 0$ and $\partial s/\partial T = 0$, this equation simplifies into

$$\frac{d\ln(a^3)}{dt} = -3H \frac{\frac{d\rho_{\text{set}}}{dT}}{\rho_{\text{set}} + P_{\text{set}}}. \quad (2.6)$$

In the standard case of this equation, since dark matter and neutrinos can be considered decoupled, the densities ρ_{set} and pressures P_{set} are given by

$$\rho_{\text{set}} = \rho_{\gamma} + \rho_{\text{b}} + \rho_{e^-} + \rho_{e^+}, \quad (2.7)$$

$$P_{\text{set}} = P_{\gamma} + P_{\text{b}} + P_{e^-} + P_{e^+}. \quad (2.8)$$

For photons, these can be computed using statistical mechanics to give

$$\rho_\gamma = \frac{\pi^2}{15} T^4, \quad P_\gamma = \frac{1}{3} \rho_\gamma, \quad (2.9)$$

and similarly, for neutrinos, to give

$$\rho_{\nu,\bar{\nu}} = N_\nu \frac{7}{8} \frac{\pi^2}{15} T_\nu^4, \quad P_{\nu,\bar{\nu}} = \frac{1}{3} \rho_{\nu,\bar{\nu}}, \quad (2.10)$$

where N_ν is the number of Standard Model neutrino species, modified by the non-exact relativistic behavior of e^\pm using the value $N_\nu = 3.046$ [11]. The neutrino temperature is related to the scale factor by

$$\frac{d \ln(a^3)}{dT_\nu} = - \frac{\frac{d\rho_{\nu,\bar{\nu}}}{dT_\nu}}{\rho_{\nu,\bar{\nu}} + P_{\nu,\bar{\nu}}} - \frac{1}{3} T^4 \mathcal{N}(T), \quad (2.11)$$

where $\mathcal{N}(T)$ is a function describing the incomplete neutrino decoupling given in Eq. (A24) of [3],

$$\mathcal{N}(T) = \begin{cases} \exp \left[\sum_{i=1}^{13} n_i \left(\frac{m_e}{T} \right)^i \right] & \text{if } m_e/T < 4, \\ 0 & \text{if } m_e/T \geq 4, \end{cases} \quad (2.12)$$

where $n_i = (-10.22, 61.24, -340.33, 1057.27, -2045.58, 2605.91, -2266.15, 1374.26, -586.06, 174.87532902234145, -35.72, 4.75, -0.37, 0.013)$, as found in [12–14]. After neutrino decoupling, the neutrino temperature is given by $T_\nu/T = (4/11)^{1/3}$.

We parameterize the sums of the e^\pm densities and pressures as

$$\rho_{e^-} + \rho_{e^+} = \frac{2}{\pi^2} m_e^4 \sum_{n=1}^{\infty} (-1)^{n+1} \cosh(n\phi_e) M(nz), \quad (2.13)$$

and

$$P_{e^-} + P_{e^+} = \frac{2}{\pi^2} m_e^4 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{nz} \cosh(n\phi_e) L(nz), \quad (2.14)$$

where we have defined the dimensionless electron mass, $z = m_e/T$, and chemical potential, $\phi_{e^-} = -\phi_{e^+} \equiv \phi_e = \mu_e/T$, and where

$$L(z) = \frac{K_2(z)}{z}, \quad \text{and} \quad M(z) = \frac{1}{z} \left(\frac{3}{4} K_3(z) + \frac{1}{4} K_1(z) \right), \quad (2.15)$$

are defined using modified Bessel functions K_i [15]. For faster computation, these convergent sums are truncated at $n = 7$.

Another constraint equation comes from charge conservation. The difference between the e^\pm densities is linked to the nuclei abundances by

$$n_{e^-} - n_{e^+} = \frac{h_\eta T^3 S}{M_u}, \quad (2.16)$$

where

$$S = \sum_i Z_i Y_i, \quad (2.17)$$

with Z_i and Y_i , the charge number and abundance of i -th nucleus respectively. h_η depends on the baryon-to-photon ratio, [2, 16, 17]

$$h_\eta(T) = M_u \frac{n_\gamma(T)}{T^3} \eta(T), \quad (2.18)$$

where M_u is the unit atomic mass and η is the baryon-to-photon ratio. The difference in Eq. (2.16) can also be expressed as

$$n_{e^-} - n_{e^+} = \frac{2}{\pi^2} m_e^3 \sum_{i=1}^{\infty} (-1)^{i+1} \sinh(n\phi_e) L(nz). \quad (2.19)$$

Using Eqs. (2.16) and (2.19), the electron chemical potential can be determined using

$$\frac{d\phi_e}{dt} = \frac{\partial\phi_e}{\partial T} \frac{dT}{dt} + \frac{\partial\phi_e}{\partial a} \frac{da}{dt} + \frac{\partial\phi_e}{\partial S} \frac{dS}{dt}. \quad (2.20)$$

Finally, the baryon density and pressure are given by the sums over the i -th nuclei [16],

$$\rho_b = h_\eta T^3 \left(1 + \sum_i \left(\frac{\Delta M_i}{M_u} + \zeta T \right) Y_i \right), \quad (2.21)$$

and

$$P_b = h_\eta T^3 \left(\frac{2}{3} \zeta T \sum_i Y_i \right), \quad (2.22)$$

where ΔM_i is the mass excess of i -th nucleus (see Table 1 in Appendix A) and $\zeta = 3/2M_u$. h_η can be determined dynamically using $h_\eta \sim \rho_b/T^3 \sim 1/a^3 T^3$, which implies the logarithmic relation

$$\frac{d \ln(h_\eta)}{dt} = -3 \left(\frac{d \ln(a)}{dt} + \frac{d \ln(T)}{dt} \right). \quad (2.23)$$

2.1.2 Nuclear reactions

The set of nuclear reactions used in `AlterBBN` is given in Tables 2–4 in Appendix A. Each one of them can be written under the generalized form (to take into account reactions where three nuclei are involved [18]),

$$N_i^{A_i} Z_i + N_j^{A_j} Z_j + N_k^{A_k} Z_k \leftrightarrow N_l^{A_l} Z_l + N_m^{A_m} Z_m + N_n^{A_n} Z_n, \quad (2.24)$$

where N_i is the number of nuclei, Z_i , that enter into the reaction and A_i is their atomic number (see Table 1 in Appendix A). Then the abundance evolution of any nuclei i is given by

$$\frac{dY_i}{dt} = N_i \sum_{j,k,l,m,n} \left(-\frac{Y_i^{N_i} Y_j^{N_j} Y_k^{N_k}}{N_i! N_j! N_k!} \Gamma_{ijk \rightarrow lmn} + \frac{Y_l^{N_l} Y_m^{N_m} Y_n^{N_n}}{N_l! N_m! N_n!} \Gamma_{lmn \rightarrow ijk} \right), \quad (2.25)$$

where $\Gamma_{ijk \rightarrow lmn}$ and $\Gamma_{lmn \rightarrow ijk}$ are the forward and reverse reaction rates respectively.

2.1.3 Initial conditions

The dynamical variables $h(t)$, $\phi_e(t)$ and $Y_i(t)$, are all functions of time, or equivalently, of temperature. The initial temperature is denoted by T_i . The initial condition for h_η depends on the initial value of the baryon-to-photon ratio, η_i , which is obtained from entropy conservation

$$h_\eta(T_i) = M_u \frac{n_\gamma(T_i)}{T_i^3} \eta_0 \left(1 + \frac{s_{e^\pm}(T_i)}{s_\gamma(T_i)} \right), \quad (2.26)$$

where η_0 is the CMB baryon-to-photon ratio. For any species, the entropy density is

$$s_{\text{sp}} = \frac{\rho_{\text{sp}} + P_{\text{sp}}}{T_{\text{sp}}}. \quad (2.27)$$

The initial condition for ϕ_e is

$$\phi_e(T_i) \approx \frac{\pi^2 h_\eta(T_i) Y_p}{2 M_u z_i^3} \frac{1}{\sum_{n=1}^{\infty} (-1)^{n+1} n L(n z_i)}, \quad (2.28)$$

where Y_p is the initial proton abundance and z_i is the initial dimensionless electron mass. The initial proton and neutron abundances Y_p and Y_n are given by the equilibrium of the reaction $p \leftrightarrow n$

$$Y_p(T_i) = \frac{1}{1 + e^{-q/T_i}}, \quad Y_n(T_i) = \frac{1}{1 + e^{q/T_i}}, \quad (2.29)$$

where $q = m_n - m_p$ is the nucleon mass difference. A similar equilibrium equation is applied to find the (small) initial deuterium abundance [17]. All the other nucleus abundances are 0 because they have not started to form yet. The initial time can be found from the initial temperature through an ‘‘infinite temperature’’ approximation [17]

$$t_i = \frac{\sqrt{12\pi G\sigma}}{T_i^2}, \quad (2.30)$$

where $\sigma = \pi^2/60$ is the Stefan-Boltzmann constant.

The set of Eqs. (2.1), (2.3), (2.20), (2.23) and (2.25) will be solved by different integration methods, as described in Appendix B.

2.2 Modified cosmologies

In this section, we present the several modified scenarios, compared to the standard cosmology BBN, that are implemented in the `AlterBBN` program.

2.2.1 Modified expansion rate

The addition of any new energy component in the early universe, such as equivalent neutrinos or any kind of effective “dark density”, has an impact on the Friedmann equation (2.1). Indeed, the total density ρ_{tot} from previous section receives a “dark” contribution ρ_{D}

$$\rho_{\text{tot}} \rightarrow \rho_{\text{tot}} + \rho_{\text{D}}, \quad (2.31)$$

$$H^2 = \frac{8\pi G}{3} (\rho_{\text{tot}} + \rho_{\text{D}}). \quad (2.32)$$

This modification affects the computation of the Hubble parameter, as shown by Eq. (2.32). For the case of a “dark density”, we have [19]

$$\rho_{\text{D}}(T) = \kappa_{\rho} \rho_{\gamma}(T_0) \left(\frac{T}{T_0} \right)^{n_{\rho}}, \quad (2.33)$$

where $T_0 = 1$ MeV is chosen as a typical energy scale to fit the BBN initial conditions. Thus, κ_{ρ} is the ratio of the dark density to the photon density at this energy scale. n_{ρ} is the decrease exponent of this dark density (4 for radiation, 3 for matter, *etc.*). A temperature cut can be added below which this density is strictly 0. If the dark fluid is in interaction with the plasma, it can enter the set of interacting components of Eq. (2.6), otherwise it only affects the expansion rate.

The pressure P_{D} associated to ρ_{D} is calculated automatically from a combination of the conservation equation and the entropy density evolution

$$P_{\text{D}} = s \frac{dT}{ds} \left(\frac{d\rho_{\text{D}}}{dT} - \rho_{\text{D}} \frac{ds}{dT} \right), \quad (2.34)$$

where s is the entropy density. The total radiation entropy, s_{rad} , is parameterized through the effective relativistic entropy degrees of freedom h_{eff} ,

$$s_{\text{rad}}(T) = h_{\text{eff}}(T) \frac{2\pi^2}{45} T^3. \quad (2.35)$$

The $h_{\text{eff}}(T)$ are tabulated in the directory `sgStar_heff`.

Another consequence of this modification is the computation of the initial time in the `AlterBBN` program, as one can see from Eq. (2.30). As long as the density is larger at a given time, the Hubble parameter is larger, and thus the time is smaller. For a fixed temperature, a correction has to be applied to the initial time t_i following

$$t_{i,\text{D}} = \frac{H_{\text{SBBN}}}{H_{\text{D}}} t_i, \quad (2.36)$$

where H_{SBBN} is the standard Hubble rate given in Eq. (2.1) and H_{D} the modified one given by Eq. (2.32).

2.2.2 Additional neutrino species

The `AlterBBN` program leaves the possibility to modify the number of Standard Model neutrino species through the value of N_ν (see Eq. (2.10)). It is possible to add equivalent neutrinos to this number through a contribution ΔN_ν , and we have $N_{\text{tot}} = N_\nu + \Delta N_\nu$.

In addition, there could be a neutrino degeneracy between the 3 neutrino species of the Standard Model, which leads to non-zero dimensionless chemical potentials ξ_{ν_1} , ξ_{ν_2} and ξ_{ν_3} , which are not necessarily equal. This will have two consequences.

The first one is a modification of Eq. (2.10) for the computation of the neutrino density. The exact statistical mechanics formula is, for each species $i = 1, 2, 3$

$$\rho_{\nu_i, \bar{\nu}_i} = \frac{1}{2\pi^2} T_\nu^4 \int_0^{+\infty} dx \frac{x^3}{1 + \exp(x \mp \xi_{\nu_i})}, \quad (2.37)$$

which for small degeneracies ($\xi_{\nu_i} < 0.3$) is approximated by the expansion [2]

$$\rho_{\nu_i} + \rho_{\bar{\nu}_i} \approx \frac{\pi^2}{15} T_\nu^4 \left(\frac{7}{8} + \frac{15}{4\pi^2} \xi_{\nu_i}^2 + \frac{15}{8\pi^4} \xi_{\nu_i}^4 + \mathcal{O}(\xi_{\nu_i}^4) \right), \quad (2.38)$$

and for high degeneracies ($\xi_{\nu_i} > 30$) by the expansion [2]

$$\rho_{\nu_i} + \rho_{\bar{\nu}_i} \approx \frac{1}{8\pi^2} (T_\nu \xi_{\nu_i})^4 \left(1 + \frac{2\pi^2}{\xi_{\nu_i}^2} + \mathcal{O}\left(\frac{1}{\xi_{\nu_i}^2}\right) \right). \quad (2.39)$$

For intermediate degeneracies ($0.3 < \xi_{\nu_i} < 30$), Eq. (2.37) has to be integrated numerically.

The second consequence is a modification of the weak interaction equilibrium in the reaction $p \leftrightarrow n$. The initial abundances of protons and neutrons, given in Eq. (2.29), become

$$Y_p(T_i) = \frac{1}{1 + e^{-q/T_i - \xi_{\bar{\nu}_e}}}, \quad Y_n(T_i) = \frac{1}{1 + e^{q/T_i + \xi_{\bar{\nu}_e}}}. \quad (2.40)$$

2.2.3 Modification of the entropy content

The early universe content can also be modified by adding entropy components, driven by particle annihilation, or simply any kind of “dark entropy” density. The relation between the scale factor and temperature is then given by Eq. (2.5), where s is the total entropy density, composed of radiation and dark entropies, denoted by s_{rad} and s_{D} . Two different cases can occur (simultaneously or separately):

1 – Dark entropy: the “dark entropy” s_{D} is a function of the temperature T , but is not linked to any reheating process of radiation, so that the term $\partial s_{\text{D}}/\partial t$ vanishes in Eq. (2.5). Two different derivations are possible. We can first express it as [20]

$$s_{\text{D}} = \kappa_s s_\gamma(T_0) \left(\frac{T}{T_0} \right)^{n_s}, \quad (2.41)$$

where the dark entropy to photon entropy ratio, κ_s , is taken at the energy scale $T_0 = 1$ MeV, and n_s is the decay exponent of this component. A temperature cut below which this density vanishes can be added.

A second derivation of the “dark entropy” is possible by injecting additional entropy Σ_D [21]

$$\Sigma_D(T) = \kappa_\Sigma \Sigma_{\text{rad}}^{\text{eff}}(T_0) \left(\frac{T}{T_0} \right)^{n_\Sigma}, \quad (2.42)$$

where κ_Σ is the ratio of the dark entropy injection to the radiation entropy density time-derivative, $\Sigma_{\text{rad}}^{\text{eff}}(T) \equiv \left| \frac{ds_{\text{rad}}}{dt} \right| = 3Hs_{\text{rad}}$ at $T_0 = 1$ MeV, and n_Σ is the decay exponent of this production. The associated “dark entropy” density is thus the integral

$$s_D(T) = 3\sqrt{\frac{5}{4\pi^3 G}} h_{\text{eff}}(T) T^3 \int_0^T dT' \frac{\sqrt{g_*(T)} \Sigma_D(T')}{h_{\text{eff}}^2(T') T'^6 \sqrt{1 + \rho_D(T)/\rho_{\text{rad}}(T)}}, \quad (2.43)$$

where the values of $h_{\text{eff}}(T)$ and $g_{\text{eff}}(T)$ are tabulated in `sgStar_heff`.

2 – Reheating: the radiation entropy s_{rad} can receive contributions from a radiation entropy injection Σ_{rad} at constant temperature, such that $\partial s/\partial T = 0$ and

$$\frac{ds_{\text{rad}}}{dt} = -3Hs_{\text{rad}} + \Sigma_{\text{rad}}, \quad (2.44)$$

which will modify the relation between the temperature and the time according to Eq. (2.5), and result in a “reheating” of the primordial plasma and a local increase of the radiation entropy density. Here, we use

$$\Sigma_{\text{rad}}(T) = \kappa_{\Sigma_r} \Sigma_{\text{rad}}^{\text{eff}}(T_0) \left(\frac{T}{T_0} \right)^{n_{\Sigma_r}}, \quad (2.45)$$

where κ_{Σ_r} is the ratio of the radiation entropy injection to the radiation entropy density time-derivative $\Sigma_{\text{rad}}^{\text{eff}}(T) \equiv \left| \frac{ds_{\text{rad}}}{dt} \right| = 3Hs_{\text{rad}}$ at $T_0 = 1$ MeV and n_{Σ_r} is the decay exponent of this production.

2.2.4 Decaying scalar field

In this scenario, a primordial scalar field is decaying, as described in [22]. Its density ρ_ϕ follows the Boltzmann equation

$$\frac{d\rho_\phi}{dt} = -nH\rho_\phi - \Gamma_\phi \rho_\phi, \quad (2.46)$$

where Γ_ϕ is the decay width of the scalar field and n the decay exponent of the scalar field density in term of the expansion factor. The scalar field decay results in radiation entropy injection at constant temperature such that $\partial s/\partial T = 0$ and

$$\frac{\partial s_{\text{rad}}}{\partial t} = -3Hs_{\text{rad}} + \frac{\Gamma_\phi \rho_\phi}{T}. \quad (2.47)$$

The decay width can be related to the reheating temperature T_{RH} through [23]

$$\Gamma_\phi = \sqrt{\frac{4\pi^3 g_{\text{eff}}(T_{\text{RH}})}{45}} \frac{T_{\text{RH}}^2}{M_{\text{P}}}, \quad (2.48)$$

where g_{eff} is the effective relativistic energy degrees of freedom tabulated in `sgStar_heff`, and M_{P} is the Planck mass.

This scenario requires two input parameters, the first one being $\tilde{\rho}_\phi$ the scalar field energy density proportion to the photon energy density at the initial temperature (~ 2.3 MeV) and the second one the reheating temperature T_{RH} .

2.2.5 WIMP scenarios

Several simple WIMP scenarios have been implemented in `AlterBBN` [18]. WIMPs are a possible solution to the dark matter problem in cosmology. WIMPs are light, weakly interacting new particles characterized by their mass, m_χ , their type (Majorana or Dirac fermion, real or complex scalar), and their coupling to Standard Model particles (neutrinos and possibly equivalents, or electromagnetic interactions).

Many of the features and calculations needed for WIMPs in `AlterBBN` are similar to those mentioned before. We must add a new WIMP density and pressure interacting with the plasma through self-annihilations. WIMPs are considered to be completely decoupled from the plasma and to be non-relativistic at the time of BBN.⁶ The WIMP density and pressure are given similarly to Eqs. (2.13) and (2.14) by [15]

$$\rho_\chi = g_\chi m_\chi^4 \sum_{n=1}^{\infty} (-1)^{\beta(n+1)} \cosh(n\phi_\chi) M(nz_\chi), \quad (2.49)$$

and

$$P_\chi = g_\chi m_\chi^4 \sum_{n=1}^{\infty} \frac{(-1)^{\beta(n+1)}}{nz_\chi} \cosh(n\phi_\chi) L(nz_\chi), \quad (2.50)$$

where g_χ is the internal number of degrees of freedom of the WIMPs (1 for a real scalar, 2 for a complex scalar, 2 for a Majorana fermion, 4 for a Dirac fermion), $z_\chi = m_\chi/T_\chi$ is their dimensionless mass, and $\phi_\chi = \mu_\chi/T_\chi$ their dimensionless chemical potential. The WIMP temperature T_χ can be T or T_ν , depending on the couplings of the WIMPs to Standard Model particles (electromagnetic or neutrinos, respectively). Also, $\beta = 0$ for bosonic WIMPs and $\beta = 1$ for fermionic WIMPs. Finally, the ‘‘cosh’’ function has to be replaced by an ‘‘exp’’ function in the case of self-conjugate particles (real scalars and Majorana fermions).

WIMPs also contribute to the entropy density of the early universe and thus the initial condition for the $h_\eta(T_i)$ variable from Eq. (2.26) has to be modified to

$$h_\eta(T_i) = M_{\text{u}} \frac{n_\gamma(T_i)}{T_i^3} \eta_0 \left(1 + \frac{s_{e^\pm}(T_i) + s_\chi(T_i)}{s_\gamma(T_i)} \right). \quad (2.51)$$

⁶More sophisticated models such as those described in [24–26] will be implemented in future versions of `AlterBBN`.

Finally, if WIMPs are coupled to neutrinos, they may dynamically modify the neutrino temperature. For details on the way these modifications alter the differential equations of BBN, we refer the reader to Section 3.2 of Ref. [18].

3 Content of the AlterBBN package

The folder `alterbbn_v2.X/` contains the nine main programs of AlterBBN:

- `stand_cosmo.c`,
- `alter_eta.c`,
- `alter_neutrinos.c`,
- `alter_neutron.c`,
- `alter_etannutau.c`,
- `alter_standmod.c`,
- `alter_reheating.c`,
- `alter_phi.c`,
- `alter_wimps.c`,

together with a `README` file, a `Makefile` file and a folder `alterbbn_v2.X/src/`. The folder `alterbbn_v2.X/src/` contains the source files:

- `bbn.c`,
- `bbnrate.c`,
- `bbn_constraints.c`,
- `general.c`,
- `cosmodel.c`,

together with the files `include.h`, `numbers.h`, `bbn.h`, `bbnrate.h` – containing the headers of all the program routines – and `Makefile`. There is also a folder `sgStar_heff/` containing numerically computed tables for $h_{\text{eff}}(T)$ and $g_{\text{eff}}(T)$, mentioned in Sections 2.2.1 and 2.2.3, and a folder `contrib/newreac/` which contains routines to include reactions and isotopes from the REACLIB database [27] into AlterBBN, as explained in Section 3.4.

3.1 Parameter structure

There are two main data structures in the program `AlterBBN`, defined in the file `include.h`. The first one is:

```
typedef struct relicparam
/* structure containing the cosmological model parameters */
{
    int entropy_model,energy_model;
    double dd0,ndd,Tdend,Tddeq; // dark density
    double sd0,nsd,Tsend; // dark entropy
    double Sigmad0,nSigmad,TSigmadend; // dark entropy injection
    double Sigmarad0,nSigmarad,TSigmaradend; // standard entropy injection
    double nt0,nnt,Tnend; // non-thermal production of relics
    int coupd; // dark fluid coupling to plasma

    double quintn2,quintn3,quintn4,quintT12,quintT23,quintT34;
    // effective quintessence model

    int phi_model; // decaying scalar field model switch
    double eta_phi,Gamma_phi,rhot_phi_Tmax,n_phi; // eta_phi = b / m_phi
    double rhot_phi0,Tphi0;
    double T_RH;
    double Sigmatildestar;
    double Sigmatildestar_max;
    double Tstdstar_max;

    double mgravitino; // gravitino mass

    double relicmass;
    int scalar;

    int solver; // switch for linear or logarithmic differential equation solver
    int beta_samples;

    double Tfo,Tmax; // Freeze out and maximal temperature

    int full_comput; // Switch to deactivate the fast freeze out temperature
    determination

    double table_eff[276][3]; // Reads values from the SgStar files
}
```

```

int use_table_rhoPD;
double table_rhoPD[2][NTABMAX];
int size_table_rhoPD;

/*-----*/
/* AlterBBN parameters */
/*-----*/

int err;
int failsafe; // Switch for the integration method
double eta0; // Initial Baryon to photon ratio
double Nnu; // Number of Neutrinos (e+- included)
double dNnu; // Number of extra neutrinos (delta N_nu)
double life_neutron, life_neutron_error; // neutron lifetime
double xinu1, xinu2, xinu3; // [e-,neutrino], [muon,neutrino],
[tau,neutrino] respectively (degeneracy parameters)
double m_chi; // Mass of WIMP
double g_chi; // dof of WIMP
double Tinit; // Initial temperature
double Tnudec; // Neutrino decoupling temperature
int wimp; // Switch to enable (1) / disable (0) wimps
int SMC_wimp; // wimp coupling to SM particles. 1 for EM,
2 for neutrino, 3 for neut. and eq. neut.
int selfConjugate; // 1/0 for self-conjugate/non-self-conjugate WIMP
int fermion;
int EM_coupled, neut_coupled, neuteq_coupled;
double fierz; // Fierz interference term from LQ sector
double B_chi; // branching ratio of WIMP DM of mass  $m_p < m_\chi < m_n$ 
to explain the tau_n anomaly
double rhob0; // current baryon density
double b_cdm_ratio; // current ratio of baryon density to cold dark
matter density
int constraints; // 1=Yp, 2=+H2/H, 3=+Li7/H, 4=+He3/H
}
relicparam;

```

and it contains all the parameters necessary to compute the BBN abundances of the elements, both in standard cosmology, and in alternative cosmologies. This structure parallels the one in `SuperIso Relic` [6–8] and so some of its parameters are not used in `AlterBBN`.

The second data structure is:

```

typedef struct errorparam
{
    int failsafe;
    int errnumber;
    double random[];
    double life_neutron;
}
errorparam;

```

and it contains parameters needed to give estimated errors linked to the computed abundances.

`AlterBBN` has different modes to compute the abundance of the elements, determined by the `failsafe` variable of the `relicparam` structure: 0 corresponds to a fast but less precise calculation, and positive values to more precise but slower calculations. In case of a very non-standard cosmological scenario, it is advisable to set `failsafe` to 6 or more. By default, the standard mode is set to 1. A description of the different modes is provided in Section B, together with the computation times and precisions in Section C.

3.2 Main routines

The main routines compiled into the library `libbbn.a` are:

- `void Init_cosmomodel(struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` initializes the `paramrelic` structure with standard BBN values. It sets the number of neutrino species to $N_{\nu} = 3.046$ (including effects from non exactly relativistic e^{\pm} [11]), the baryon-to-photon ratio to $\eta_{a0} = 6.10 \times 10^{-10}$ [28], the initial temperature to $T_{\text{init}} = 27 \times 10^9$ K (corresponding to 2.3 MeV, an adequate value before the real start of BBN) and the lifetime of the neutron $\text{life_neutron} = 880.2$ s [28] (with its associated error to $\text{life_time_error} = 1.0$ s). All the other parameters are set to 0.

- `void Init_cosmomodel_param(double eta, double Nnu, double dNnu, double life_neutron, double life_neutron_error, double xinu1, double xinu2, double xinu3, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies some parameters of the `paramrelic` structure with potentially non-standard values: the baryon-to-photon ratio η_{a0} , the number of Standard Model neutrino species N_{ν} , the number of additional neutrino species dN_{ν} , degeneracy of the Standard Model neutrinos

xinu1, xinu2 and xinu3 (see Section 2.2.2) and finally, the neutron lifetime `life_neutron` and the associated error `life_neutron_error`.

- `void Init_dark_density(double dd0, double ndd, double T_end, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark density described in Section 2.2.1. Here $\kappa_\rho = dd0$, $n_\rho = ndd$ and `T_end` is the temperature cutoff at which the effective dark density is set to 0.

- `void Init_dark_entropy(double sd0, double nsd, double T_end, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark entropy density, in the case of no reheating, as described in Section 2.2.3. Here, $\kappa_s = sd0$, $n_s = nsd$, and `T_end` is the temperature cutoff at which the effective dark entropy is set to zero.

- `void Init_dark_entropySigmaD(double Sigmad0, double nSigmad, double T_end, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark entropy production in the no-reheating case described in Section 2.2.3. Here, $\kappa_\Sigma = Sigmad0$, $n_\Sigma = nSigmad$, and `T_end` is the temperature cutoff at which the effective entropy production is set to zero.

- `void Init_entropySigmarad(double Sigmarad0, double nSigmarad, double T_end, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the radiation entropy production in the reheating case, described in Section 2.2.3. Here $\kappa_{\Sigma_r} = Sigmarad0$, $n_{\Sigma_r} = nSigmarad$ and `T_end` is the cutoff temperature at which the radiation entropy production is set to 0.

- `void Init_scalarfield(double rhotilde_phi, double T_RH, double eta_phi, double n_phi, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the decay of a scalar field during BBN, as described in Section 2.2.4. Here, `rhotilde_phi` is the ratio of the scalar field density over the

photon density at initial temperature, `T_RH` is the reheating temperature, and `n_phi` is the decay exponent of the scalar field density. The parameter `eta_phi` has no effect in `AlterBBN` and is set to zero.

- `void Init_wimp(double mass_wimp, int EM_coupled, int neut_coupled, int neuteq_coupled, int fermion, int selfConjugate, double g_chi, struct relicparam* paramrelic)`

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the existence of WIMPs during BBN, as described in Section 2.2.5. Here, the parameters related to WIMP injection are the WIMP mass, $m_\chi = \text{mass_wimp}$, the Standard Model couplings, `EM_coupled`, `neut_coupled`, and `neuteq_coupled` (all switches between 0/1 for inactive/active), and the type of WIMP particle, `fermion` and `selfConjugate` (both switches between 0/1 which represent the four types of wimps described in paragraph 2.2.5).

- `void rate_weak(double f[], struct relicparam* paramrelic, struct errorparam* paramerror)`

This routine defined in `bbnrate.c` computes the forward reaction rates of the β -decays corresponding to the processes (2–11) given in Table 2 in Appendix A and stores them into slots (2–11) of the array `f[]`. There is no reverse reaction so slots (2–11) of the array `r[]` do not need to be computed.

- `void rate_pn(double f[], double r[], double T9, double Tnu, struct relicparam* paramrelic, struct errorparam* paramerror)`

This routine defined in `bbnrate.c` computes the forward and reverse reaction rates of the nuclear reaction 1 (proton-neutron conversion) given in Table 2 in Appendix A and stores them into the slot (1) of the arrays `f[]` and `r[]`.

- `void rate_all(double f[], double T9, struct relicparam* paramrelic, struct errorparam* paramerror)`

This routine defined in `bbnrate.c` computes the forward reaction rates of the nuclear reactions (12-100) given in Tables 2 and 3 in Appendix A and stores them into slots (12-100) of array `f[]` (the reverse reaction rates will be estimated elsewhere using detailed balance factors contained in the table `reacparam[][]` and stored in the array `r[]`).

- `int nucl(struct relicparam* paramrelic, double ratioH[])`

This routine defined in `bbn.c` is the main routine of the program, as it computes the BBN abundance ratios of all nuclei given in Table 1 in Appendix A and in particular, the light elements $Y_p = \rho(^4\text{He})/\rho_b$, $\text{H2_H} = [^2\text{H}]/[\text{H}]$, $\text{He3_H} = [^3\text{He}]/[\text{H}]$, $\text{Li7_H} = [^7\text{Li}]/[\text{H}]$, $\text{Li6_H} = [^6\text{Li}]/[\text{H}]$ and $\text{Be7_H} = [^7\text{Be}]/[\text{H}]$ (note that `He3_H` and `Li7_H` contain the contributions of post-BBN decays of respectively `H3_H` and `Be7_H`). It returns 0 if the computation succeeded or 1 otherwise.

- `int bbn_excluded(struct relicparam* paramrelic)`

This routine defined in `bbn_constraints.c` is a “container” function that calls the `nucl` routine and compares its results with BBN observational constraints summarized in Appendix D. It returns 0 if the constraints are satisfied, 1 if the abundances are not compatible with the observations and -1 if the computation fails.

3.3 Error & correlations

`AlterBBN` includes BBN nuclei abundance calculation error estimates. These estimates rely on the parameters `err` and `life_neutron_error` contained in the `relicparam` structure, and the parameters `errnumber` and `random[]` in the `errorparam` structure (see Section 3.1), as well as estimated errors on the nuclear reaction rates.

The `err` parameter switches between five methods of evaluation of the abundances of the elements and their errors:

- `err = 0`: central values of the nuclear reaction rates are used for all reactions,
- `err = 1`: higher values are used for all reactions,
- `err = 2`: lower values are used for all reactions,
- `err = 3`: the covariance matrix is calculated through the variation of the parameters, using the higher value of the reaction rates and following the method of Ref. [10],
- `err = 4`: randomly Gaussian distributed values (between lower and higher) are used for all reactions.

The nine main programs listed in the next section run successively the `err = 2`, `err = 0` and `err = 1` types in order to give associated “lower”, central and “upper” values of the abundances. Then they run the `err = 3` type for all the reactions in order to compute a correlation matrix between the abundances, stored in the table `corr_ratioH[] []`.

An additional run of the `err = 4` type is implemented to perform a Monte Carlo correlation analysis, but it is commented out due to the extensive computational

time (the abundance computation has to be done a lot of times, a number defined by the variable `niter = 1000`, by default).

The `err = 0` type is used in order to compare the computed abundances to the most recent observations using a χ^2 analysis (`chi2 without correlations`). The `err = 3` type is used to compute the correlations and obtain a χ^2 (`chi2 including correlations`).

3.4 REACLIB reactions

By default, `AlterBBN` incorporates 26 elements and 100 nuclear reactions. A new module has been added in the latest version to easily include more elements and reactions from the JINA REACLIB database [27]. The routines are contained in the directory `src/contrib/newreac/`. To generate a new set of isotopes and reactions, the program has to be compiled with `make`, which creates `create_network.x`, and run with three parameters and an optional one: name of the WINVN file, name of the REACLIB database file, maximal atomic mass to be kept, and the optional parameter can be 0, 1 or 2, which selects all the isotopes with an atomic mass smaller than the maximal value, removes the isotopes with very small abundances after BBN, or removes the isotopes with very small abundances during BBN, respectively. The program generates three files, `numbers.h`, `bbn.h` and `bbnrate.h`, which contain the REACLIB parameters and are symbolically linked into `src/`. In the packages, the latest WINVN and REACLIB files from <http://reaclib.jinaweb.org/> are included. Running

```
./create_network.x winvn_v2.0.dat results02200820.dat 30 1
```

adds 19 elements and 101 nuclear reactions. This set of new reactions is included by default in `AlterBBN`, but can be easily modified by running `create_network.x`.

4 Compilation and installation instructions

`AlterBBN` has been written in C respecting the C99 standard, and it has been tested with the GNU C and the Intel C compilers on Linux, Windows (using `Cygwin64`) and Mac. The package can be downloaded at the address:

<https://alterbbn.hepforge.org/>

The package should be unpacked in the desired directory, creating the main directory

`alterbbn_vX.X/`

containing the material described in Section 3. If needed, the user's C compiler information and flags can be specified in the file `Makefile` in this main directory. In particular, some of the computations are made in parallel using the `OpenMP` library. The user should comment out the corresponding lines if this library is not installed. More information is provided in the `README` file.

To activate the extra REACLIB reactions in `AlterBBN` (see Section 3.4), the line `#define REACLIB` has to be uncommented in `include.h`. If the number of extra elements is large, segmentation faults can occur because of the limited stack size. To circumvent the problem, it can be necessary to run `ulimit -s unlimited` before running the code.

To compile the library `libbbn.a`, type `make` in the main folder. The library file will be created in the subfolder `src/`. To compile a specific program, type `make <name>` or `make <name>.c` in the main folder, where `<name>` can be:

- `stand_cosmo` (see Section 5.1),
- `alter_eta` (see Section 5.2.1),
- `alter_neutrinos` (see Section 5.2.3),
- `alter_etannutau` (see Section 5.2.2),
- `alter_standmod` (see Section 5.3),
- `alter_reheating` (see Section 5.4),
- `alter_phi` (see Section 5.5),
- `alter_wimps` (see Section 5.6).
- `alter_neutron` (see Section 5.7).

5 Input and output description

In this section we give input and output instructions for the nine programs listed in Section 4.

5.1 Standard cosmology

The program `stand_cosmo.x` computes the BBN abundances of the nuclei as well as the associated errors and correlation matrix in the standard cosmological model. It takes one integer as argument; if it is 0, a fast calculation is performed, larger values provide slower, but more precise, calculations. Descriptions of the possible integration methods are given in Sections B and C. The values of the baryon-to-photon ratio, the neutron lifetime and the number of neutrinos species are set by the `Init_cosmomodel` routine. Running the program with:

```
./stand_cosmo.x 3
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
low:	2.474e-01	2.526e-05	1.025e-05	5.028e-10	1.689e-15	4.745e-10
cent:	2.473e-01	2.463e-05	1.034e-05	5.376e-10	1.085e-14	5.087e-10
high:	2.473e-01	2.404e-05	1.044e-05	5.746e-10	3.522e-14	5.454e-10

With uncertainties:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.473e-01	2.463e-05	1.034e-05	5.376e-10	1.085e-14	5.087e-10
+/- :	3.201e-04	3.769e-07	1.644e-07	3.517e-11	1.085e-14	3.427e-11

Correlation matrix:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
Yp	1.000000	-0.011847	0.008393	0.032030	0.001444	0.031116
H2/H	-0.011847	1.000000	-0.765797	-0.356718	0.060103	-0.369486
He3/H	0.008393	-0.765797	1.000000	0.357189	-0.006905	0.368957
Li7/H	0.032030	-0.356718	0.357189	1.000000	-0.024687	0.996535
Li6/H	0.001444	0.060103	-0.006905	-0.024687	1.000000	-0.025486
Be7/H	0.031116	-0.369486	0.368957	0.996535	-0.025486	1.000000

Compatible with BBN constraints (chi2 without correlations)

Compatible with BBN constraints (chi2 including correlations)

5.2 Standard cosmology with modified parameters

5.2.1 Modification of the baryon-to-photon ratio

The program `alter_eta.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified value of the baryon-to-photon ratio, η_0 , taken as an input argument. The next argument is optional and specifies the integration method. Running the program with:

```
./alter_eta.x 3e-10
```

returns (hereafter, only the most relevant part of the output will be given):

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.397e-01	7.545e-05	1.618e-05	1.318e-10	3.165e-14	6.869e-11
+/- :	3.195e-04	1.289e-06	1.726e-07	1.100e-11	3.134e-14	5.272e-12

Excluded by BBN constraints (chi2 without correlations)

Excluded by BBN constraints (chi2 including correlations)

5.2.2 Modifications of the baryon-to-photon ratio, neutrino number and neutron lifetime

The program `alter_etannutau.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified value of the baryon-to-photon ratio, number of Standard Model neutrino species, number of additional

neutrino species, neutron lifetime, the optional integration method choice and an optional neutron lifetime error given in seconds, taken as input arguments in respective order. Running the program with:

```
./alter_etannutau.x 6.09e-10 3.05 0.1 880.5 32 2.
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.488e-01	2.512e-05	1.041e-05	5.282e-10	1.115e-14	4.987e-10
+/- :	6.459e-03	5.415e-07	1.724e-07	3.549e-11	1.115e-14	3.430e-11

Compatible with BBN constraints (chi2 without correlations)

Compatible with BBN constraints (chi2 including correlations)

5.2.3 Modifications of the neutrino number and degeneracies

The program `alter_neutrinos.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified number of Standard Model neutrino species, number of additional neutrino species and possibly three neutrino degeneracies ξ_{ν_i} (see Section 2.2.2) as well as the optional integration method choice, taken as input arguments in respective order. Running the program with:

```
./alter_neutrinos.x 3.046 0.1 0.1 0.1 0.1
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.255e-01	2.354e-05	1.019e-05	5.046e-10	9.207e-15	4.799e-10
+/- :	2.965e-04	4.256e-07	1.678e-07	3.268e-11	9.181e-15	3.180e-11

Compatible with BBN constraints (chi2 without correlations)

Excluded by BBN constraints (chi2 including correlations)

5.3 Modified expansion rate and entropy content

The program `alter_standmod.x` computes the BBN abundances, errors and correlations in a cosmology scenario without reheating where the expansion rate and the entropy content are modified by the injection of a dark component throughout the BBN epoch (see Section 2.2.3). It takes 4–8 ordered arguments, κ_ρ , n_ρ , κ_s and n_s , and an optional switch to specify if the dark energy is coupled to the plasma (if set to 1, the dark energy enters the `set` in Eq. (2.6), otherwise it only modifies the expansion rate), the cutoff temperatures in MeV for dark energy and dark entropy, and the integration method. Running the program with:

```
./alter_standmod.x 0.1 3 0.1 4 0 0. 0.
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.652e-01	4.249e-05	1.197e-05	3.244e-10	2.007e-14	2.754e-10
+/- :	3.243e-04	1.343e-06	1.978e-07	2.380e-11	1.996e-14	2.305e-11

Excluded by BBN constraints (chi2 without correlations)

Excluded by BBN constraints (chi2 including correlations)

5.4 Effective reheating scenario

The program `alter_reheating.x` computes the BBN abundances, errors and correlations in a cosmology scenario with reheating where dark energy and entropy are injected throughout the BBN epoch. It takes 5–10 ordered arguments, κ_ρ , n_ρ , κ_{Σ_r} , n_{Σ_r} , the temperature cutoff for the injection in MeV and optionally, κ_s , n_s , κ_Σ and n_Σ (see Section 2.2.3), the integration method, and the initial value of η . Running the program with:

```
./alter_reheating 0 0 1 6 0.01
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	1.603e-01	3.469e-04	3.224e-05	2.222e-10	9.056e-14	1.662e-12
+/- :	2.730e-04	4.347e-06	2.825e-07	4.125e-11	8.706e-14	1.157e-13

Excluded by BBN constraints (chi2 without correlations)

Excluded by BBN constraints (chi2 including correlations)

5.5 Decaying scalar field scenario

The program `alter_phi.x` computes the BBN abundances, errors and correlations in a cosmological scenario where a scalar field is decaying throughout the BBN epoch. It takes two arguments, the scalar field density ratio $\tilde{\rho}_\phi$, and the reheating temperature, T_{RH} , in MeV (see Section 2.2.4), and three more optional arguments, the scalar field decay exponent (3 by default, matter-like behaviour), the initial temperature in MeV, the integration method, and the initial value of η . Running the program with:

```
./alter_phi.x 0.1 10
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.454e-01	2.532e-05	1.040e-05	5.156e-10	1.104e-14	4.865e-10
+/- :	3.202e-04	1.040e-06	1.894e-07	4.650e-11	1.093e-14	4.596e-11

Compatible with BBN constraints (chi2 without correlations)

Compatible with BBN constraints (chi2 including correlations)

5.6 WIMP scenario

The program `alter_wimps.x` computes the BBN abundances, errors and correlations in a cosmological scenario where WIMPs are added to the Standard Model particles. It takes three arguments, which are the type of WIMP particle (1: real scalar, 2: complex scalar, 3: Majorana fermion, 4: Dirac fermion), the type of coupling to the Standard Model particles (1: neutrinos, 2: electromagnetic, 3: equivalent neutrinos) and m_χ (to be given in MeV) (see Section 2.2.5). An additional parameter can be

given to specify the integration method. Running the program with:

```
./alter_wimp.x 2 2 15.
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.473e-01	2.421e-05	1.028e-05	5.496e-10	1.067e-14	5.212e-10
+/- :	3.183e-04	5.985e-07	1.712e-07	3.839e-11	1.065e-14	3.800e-11

Compatible with BBN constraints (chi2 without correlations)

Compatible with BBN constraints (chi2 including correlations)

5.7 Neutron decay scenarios

The program `alter_neutron.x` computes the BBN abundances, errors and correlations in a cosmological scenario where neutron beta decay is modified by beyond the Standard Model physics such as tensor or scalar currents, or dark decay channels with WIMPs near the neutron mass.

The reaction rates require phase space integrals with fermion state occupancy terms $1/(1 + e^{\pm x/z})$, the electron occupancy factor, where $x = E_e/m_e$ is the reduced electron energy, and $z = T_9 k_B/m_e$ is the dimensionless reduced final-state temperature, and $1/(1 + e^{\pm x_\nu/z_\nu \pm \xi_{\nu e}})$, the neutrino occupancy factor, where $x_\nu = q - x$ is the reduced neutrino energy, and $z_\nu = T_\nu k_B/m_e$ is the dimensionless reduced neutrino temperature, with $q = (m_n - m_p)/m_e \approx 2.53101$, the dimensionless neutron endpoint. `alter_neutron.x` increases the precision of the phase-space by adding the Fermi function that corrects for the electrostatic interaction of protons and betas,

$$F(\pm\eta) = \frac{\pm\eta}{1 \pm e^{\pm\eta}}; \quad \eta = 2\pi\alpha/\beta. \quad (5.1)$$

It also allows for the careful calculation of scenarios with an added scalar or tensor interaction with the Fierz interference parameter for the free neutron, b_n , with the variable `fierz`, and a electron neutrino chemical potential $\xi_{\nu e}$ using `xinu1`, as discussed in Section 2.2.1.

The first of the four integrals calculated is just the standard neutron decay familiar from low energy, but with and added Fierz term,

$$\Gamma_{n \rightarrow pe\bar{\nu}} = \tilde{\Gamma}_0 \int_1^\infty F(\eta) f(-x/z) f(-x_\nu/z_\nu - \xi_{\nu e}) \beta x_\nu^2 x (x + b_n) dx, \quad (5.2)$$

where $f(x) = 1/(1 + e^x)$ and $\tilde{\Gamma}_0$ is the phase space integral modified neutron decay rate at low temperature,

$$(\tau_n \tilde{\Gamma}_0)^{-1} = \int_1^q F(\eta) \beta x_\nu^2 x (x + b_n) dx \approx 1.69174 + 1.10855 b_n, \quad (5.3)$$

We then have the rates for the absorption of a positron or an electron neutrino

$$\Gamma_{n\bar{e} \rightarrow p\nu} = \tilde{\Gamma}_0 \int_1^\infty f(x/z) f(-x_\nu/z_\nu - \xi_{\nu e}) \beta x_\nu^2 x (x - b_n) dx, \quad (5.4)$$

and

$$\Gamma_{n\nu\rightarrow pe} = \tilde{\Gamma}_0 \int_1^\infty F(-\eta) f(-x/z) f(x_\nu/z_\nu + \xi_{\nu_e}) \beta x_\nu^2 x(x - b_n) dx, \quad (5.5)$$

where $x_\nu = x + q$. We also have the reverse reaction $p + e^- \rightarrow n + \nu$ which is [29]

$$\Gamma_{pe\rightarrow n\nu} = \tilde{\Gamma}_0 \int_1^\infty F(\eta) f(x/z) f(x_\nu/z_\nu + \xi_{\nu_e}) \beta x_\nu^2 x(x + b_n) dx, \quad (5.6)$$

where again as for the decay case, here $x_\nu = q - x$. When `alter_neutron.x` computes these integrals, it performs a numerical sum using the number of samples specified by the `beta_samples` variable. Hundreds to thousands of samples are needed to probe alterations to beta decay parameters.

```
./alter_neutron.x 880.2 1.0
```

returns:

	Yp	H2/H	He3/H	Li7/H	Li6/H	Be7/H
value:	2.483e-01	2.441e-05	1.032e-05	5.471e-10	1.081e-14	5.183e-10
+/- :	3.185e-04	6.798e-07	1.816e-07	3.902e-11	1.079e-14	3.860e-11

Compatible with BBN constraints (chi2 without correlations)

Compatible with BBN constraints (chi2 including correlations)

6 Example of results

To illustrate the capabilities of `AlterBBN`, we investigate the BBN constraints on the dark density and reheating scenarios, described in Sections 2.2.1 and 5.4, respectively.

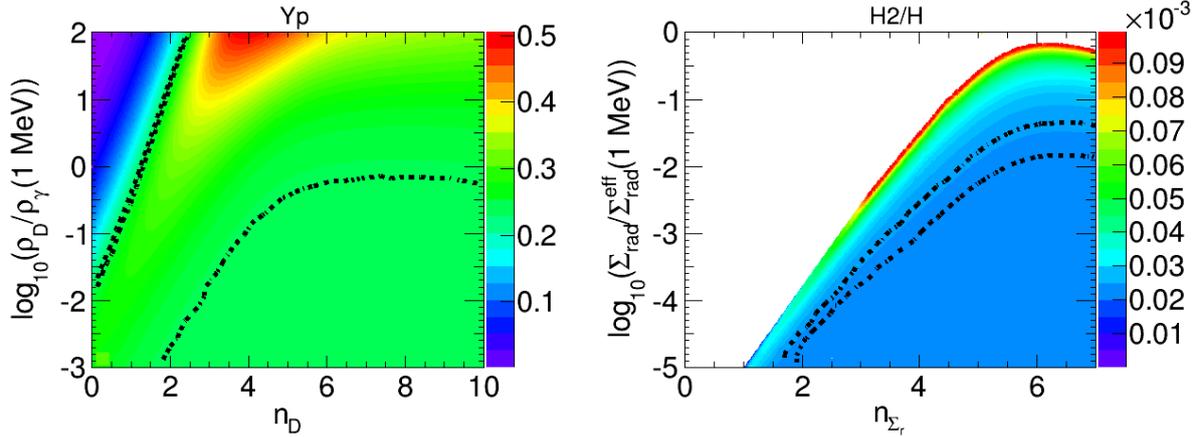


Figure 1. Y_p in the dark density scenario (left) and $[^2\text{H}]/[\text{H}]$ in the reheating scenario (right). The parameter regions compatible with the BBN constraints are for the dark density scenario (left) in between the two left lines and below the curve on the right, and for the reheating scenario (right) in between the two curves. The color corresponds to the values of Y_p and $[^2\text{H}]/[\text{H}]$.

For the first case, we can constrain the value of the the dark density at $T = 1$ MeV and its exponent, and for the second case, we can constrain the value of the radiation injection at $T = 1$ MeV and its exponent. We used `failsafe=3` for an improved precision. The results are shown in Fig. 1. The color scale shows the value of Y_p in the dark density scenario, and of $[\text{H}]/[\text{H}]$ in the reheating scenario. The black lines are the limits obtained using the constraints described in Appendix D.

A Nuclear reaction network

Table 1. Table summarizing the conventions used to denote the nuclei. Mass excess is given in MeV.

index	nuclei	atomic number	charge number	mass excess
1	n	1	0	8.071388
2	p	1	1	7.289028
3	^2H	2	1	13.135825
4	^3H	3	1	14.949915
5	^3He	3	2	14.931325
6	^4He	4	2	2.424931
7	^6Li	6	3	14.0864
8	^7Li	7	3	14.9078
9	^7Be	7	4	15.7696
10	^8Li	8	3	20.9464
11	^8B	8	5	22.9212
12	^9Be	9	4	11.34758
13	^{10}B	10	5	12.05086
14	^{11}B	11	5	8.6680
15	^{11}C	11	6	10.6506
16	^{12}B	12	5	13.3690
17	^{12}C	12	6	0
18	^{12}N	12	7	17.3382
19	^{13}C	13	6	3.125036
20	^{13}N	13	7	5.3455
21	^{14}C	14	6	3.019916
22	^{14}N	14	7	2.863440
23	^{14}O	14	8	8.006521
24	^{15}N	15	7	0.101439
25	^{15}O	15	8	2.8554
26	^{16}O	16	8	-4.737036

B Description of the integration methods

The integration method can be modified using the `failsafe` parameter of the `relicparam` structure.

B.1 Linearization

The abundance evolution of any nuclei i is given by the Boltzmann equation

$$\frac{dY_i}{dt} = N_i \sum_{j,k,l,m,n} \left(-\frac{Y_i^{N_i} Y_j^{N_j} Y_k^{N_k}}{N_i! N_j! N_k!} \Gamma_{ijk \rightarrow lmn} + \frac{Y_l^{N_l} Y_m^{N_m} Y_n^{N_n}}{N_l! N_m! N_n!} \Gamma_{lmn \rightarrow ijk} \right). \quad (\text{B.1})$$

Unfortunately, the system for all the nuclei is composed of highly non-linear equations, and requires special attention. The system of equations has to be linearized, and written in the form

$$\frac{d\tilde{\mathbf{Y}}}{dt} = M(Y_i) \tilde{\mathbf{Y}}, \quad (\text{B.2})$$

where $\tilde{\mathbf{Y}} = (\mathbf{Y}_1, \dots, \mathbf{Y}_n)$ and M is a matrix depending on the Y_i . The matrix elements can be obtained with a comparison to

$$\begin{aligned} \frac{d\mathbf{Y}_i}{dt} = N_i \sum_{j,k,l,m,n} & \left[-\frac{1}{N_i! N_j! N_k! (N_i + N_j + N_k)} \Gamma_{ijk \rightarrow lmn} \right. \\ & \times \left[N_i Y_i^{N_i-1} Y_j^{N_j} Y_k^{N_k} \mathbf{Y}_i + N_j Y_i^{N_i} Y_j^{N_j-1} Y_k^{N_k} \mathbf{Y}_j + N_k Y_i^{N_i} Y_j^{N_j} Y_k^{N_k-1} \mathbf{Y}_k \right] \\ & + \frac{1}{N_l! N_m! N_n! (N_l + N_m + N_n)} \Gamma_{lmn \rightarrow ijk} \\ & \left. \times \left[N_l Y_l^{N_l-1} Y_m^{N_m} Y_n^{N_n} \mathbf{Y}_l + N_m Y_l^{N_l} Y_m^{N_m-1} Y_n^{N_n} \mathbf{Y}_m + N_n Y_l^{N_l} Y_m^{N_m} Y_n^{N_n-1} \mathbf{Y}_n \right] \right]. \end{aligned} \quad (\text{B.3})$$

B.2 Stiff equation integration

In a discrete integration, Eq. (B.2) becomes

$$\frac{\tilde{\mathbf{Y}}_{n+1} - \tilde{\mathbf{Y}}_n}{\Delta t} = M(Y_n) \tilde{\mathbf{Y}}_n, \quad (\text{B.4})$$

where n denotes the integration step number and Δt the step-size. This equation can be rewritten as

$$\tilde{\mathbf{Y}}_{n+1} = [1 + M(Y_n) \Delta t] \tilde{\mathbf{Y}}_n. \quad (\text{B.5})$$

Unfortunately, M has negative eigenvalues, and there is a high risk during the integration that the $(1 + M \Delta t)$ cancels. For this reason, we instead integrate using

$$\tilde{\mathbf{Y}}_{n+1} = [1 - M(Y_n) \Delta t]^{-1} \tilde{\mathbf{Y}}_n, \quad (\text{B.6})$$

which is well-behaved and leads to a better convergence even for sufficiently large values of Δt . The linearization is therefore applied to the matrix $[1 - M(Y_n) \Delta t]$. To obtain the values of the derivatives of Y_i , $\tilde{\mathbf{Y}}_{n+1}$ is obtained via a triangularization of the matrix with a Cholesky decomposition, and inversion with a Gaussian elimination and back substitution. The derivatives of the abundances are finally given by $(\tilde{\mathbf{Y}}_{n+1} - \tilde{\mathbf{Y}}_n)/\Delta t$.

B.3 Runge-Kutta of order 2

Methods 0–3 use a Runge-Kutta of order 2 integration,

$$\tilde{\mathbf{Y}}_{\mathbf{n}+1} = \tilde{\mathbf{Y}}_{\mathbf{n}} + \frac{d\tilde{\mathbf{Y}}}{dt} \Delta t, \quad (\text{B.7})$$

where

$$\frac{d\tilde{\mathbf{Y}}}{dt} = \frac{1}{2} \left[\frac{d\tilde{\mathbf{Y}}}{dt}(t_n) + \frac{d\tilde{\mathbf{Y}}}{dt}(t_n + \Delta t) \right]. \quad (\text{B.8})$$

Following [2], the step-size is adapted with

$$\Delta t = \min \left(\left| \frac{T}{dT/dt} \right| c_t, \left| \frac{Y_i}{dY_i/dt} \left[1 + \left(\frac{\log(Y_i)}{\log(Y_{\min})} \right) \right] \right| c_y \right), \quad (\text{B.9})$$

if $\Delta t > \Delta t_{\min}$, where $Y_{\min} = 10^{-30}$, and c_t , c_y and Δt_{\min} are given in:

Method (failsafe)	c_t	c_y	$\Delta t_{\min}(\text{s})$
0 (fastest)	0.1	0.5	10^{-2}
1 (default)	0.01	0.25	10^{-10}
2	0.005	0.1	10^{-10}
3 (slowest)	0.001	0.05	10^{-10}

These methods are the fastest ones, but lack a more robust convergence test in scenarios very far from the standard one, for which the other methods are preferred.

B.4 Runge-Kutta of order 2 with half step test

Methods 5–7 use Runge-Kutta integration of order 2, but the convergence test is different. For each step, the variable are computed twice, once with a step of size Δt , and the second with two step sizes $\Delta t/2$. If the variable values differ by more than a tolerance **prec**, the step-size is divided by 2 and the calculations start over. Otherwise, the step-size is obtained by

$$\Delta t \rightarrow 1.8 \times \min[1, \max(0.3, \text{minprec})] \times \Delta t, \quad (\text{B.10})$$

where

$$\text{minprec} = \min \left(\left| \frac{\text{prec} \times \text{variable (2 steps)}}{\text{variable (2 steps)} - \text{variable (1 step)}} \right| \right), \quad (\text{B.11})$$

where only the abundances larger than Y_{\min} are considered in the test, in addition to the other physical variables.

For the different methods, the values of Y_{\min} and the tolerances are given in:

Method (failsafe)	Y_{\min}	Tolerance prec
5 (fastest)	10^{-25}	5%
6 (recommended)	10^{-30}	1%
7 (slowest)	10^{-30}	0.1%

B.5 Runge-Kutta of order 4 with half step test

For one time step, the standard Runge-Kutta of order 4 corresponds to

$$y_{n+1} = y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad (\text{B.12})$$

where

$$k_1 = \frac{dy}{dt}(t_n, y_n), \quad (\text{B.13})$$

$$k_2 = \frac{dy}{dt}\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2}k_1\right) \quad (\text{B.14})$$

$$k_3 = \frac{dy}{dt}\left(t_n + \frac{\Delta t}{2}, y_n + \frac{\Delta t}{2}k_2\right) \quad (\text{B.15})$$

$$k_4 = \frac{dy}{dt}(t_n + \Delta t, y_n + \Delta t k_1). \quad (\text{B.16})$$

The step-size is then adapted similarly to the method of Section B.4. For the different methods, the values and tolerances of Y_{\min} are:

Method (failsafe)	Y_{\min}	Tolerance
10 (fastest)	10^{-25}	5%
11 (recommended)	10^{-30}	1%
12 (slowest)	10^{-30}	0.1%

B.6 Runge-Kutta of order 4–5

For one time step, Δt , the explicit Runge-Kutta methods correspond to

$$y_{n+1} = y_n + \Delta t \sum_{i=1}^s b_i k_i, \quad (\text{B.17})$$

where s is the number of sub-steps and

$$k_1 = \frac{dy}{dt}(t_n, y_n), \quad (\text{B.18})$$

$$k_2 = \frac{dy}{dt}(t_n + c_2 \Delta t, y_n + \Delta t(a_{21}k_1)),$$

$$k_3 = \frac{dy}{dt}(t_n + c_3 \Delta t, y_n + \Delta t(a_{31}k_1 + a_{32}k_2)), \quad (\text{B.19})$$

\vdots

$$k_s = \frac{dy}{dt}(t_n + c_s \Delta t, y_n + \Delta t(a_{s1}k_1 + a_{s2}k_2 + \dots + a_{s,s-1}k_{s-1})).$$

The methods of order 4–5 consist of evaluating y_{n+1} using simultaneously 4 and 5 sub-steps with common c_i and a_{ij} , and use the two results to estimate the numerical error and adapt the step-size.

In `AlterBBN`, if the difference is smaller than the tolerance `prec`, the step-size is adjusted to

$$\Delta t \rightarrow \min\left(1.1, \max\left(2, 0.84 (\text{prec} \times \text{minprec})^{1/4}\right)\right) \Delta t, \quad (\text{B.20})$$

otherwise,

$$\Delta t \rightarrow \max\left(0.9, \min\left(0.5, 0.84 (\text{prec} \times \text{minprec})^{1/4}\right)\right) \Delta t, \quad (\text{B.21})$$

with

$$\text{minprec} = \min\left(\left|\frac{\text{prec} \times \text{variable (2 steps)}}{\text{variable (2 steps)} - \text{variable (1 step)}}\right|\right), \quad (\text{B.22})$$

where only the abundances larger than Y_{\min} are considered in the test, in addition to the other physical variables.

B.6.1 Fehlberg-Runge-Kutta method

The parameters c_i for this method are:

i	1	2	3	4	5
c_i	1/4	3/8	12/13	1	1/2

The a_{ij} are:

a_{ij}	1	2	3	4	5
1	1/4				
2	3/32	9/32			
3	1932/2197	-7200/2197	7296/2197		
4	439/216	-8	3680/513	-845/4104	
5	-8/27	2	-3544/2565	1859/4104	-11/40

The order 4 solution is computed with:

i	1	2	3	4	5
b_i	25/216	0	1408/2565	2197/4104	-1/5

and the order 5 with:

i	1	2	3	4	5	6
b_i	16/135	0	6656/12825	28561/56430	-9/50	2/55

The other `AlterBBN` parameters are, depending on `failsafe`:

Method (<code>failsafe</code>)	Y_{\min}	Tolerance
20 (fastest)	10^{-25}	5%
21 (recommended)	10^{-30}	1%
22 (slowest)	10^{-30}	0.1%

B.6.2 Cash-Karp-Runge-Kutta method

The parameters c_i for this method are:

i	1	2	3	4	5
c_i	1/5	3/10	3/5	1	7/8

The a_{ij} are:

a_{ij}	1	2	3	4	5
1	1/5				
2	3/40	9/40			
3	3/10	-9/10	6/5		
4	-11/54	5/2	-70/27	35/27	
5	1631/55296	175/512	575/13824	44275/110592	253/4096

The order 4 solution is computed with:

i	1	2	3	4	5	6
b_i	2825/27648	0	18575/48384	13525/55296	277/14336	1/4

and the order 5 with:

i	1	2	3	4	5	6
b_i	37/378	0	250/621	125/594	0	512/1771

The other `AlterBBN` parameters are, depending on `failsafe`:

Method (<code>failsafe</code>)	Y_{\min}	Tolerance
30 (fastest)	10^{-25}	10^{-2}
31 (recommended)	10^{-30}	10^{-4}
32 (slowest)	10^{-30}	10^{-5}

C Comparison of the integration methods

In this section, we compare the different integration methods. The Runge-Kutta 4 method with `failsafe` = 12 is by construction the most precise (and slowest), to which the results will be compared.

In the standard cosmological model, we compare the precision of one single BBN calculation:

failsafe	$Y_p(\times 10^{-1})$	deviation	${}^2\text{H}/\text{H}(\times 10^{-5})$	deviation	${}^7\text{Li}/\text{H}(\times 10^{-10})$	deviation
0	2.462	-0.010	2.343	-0.120	5.680	+0.310
1	2.472	0.000	2.431	-0.032	5.473	+0.103
2	2.472	0.000	2.454	-0.009	5.402	+0.032
3	2.472	0.000	2.459	-0.004	5.382	+0.012
5	2.524	+0.048	2.672	+0.209	4.974	-0.396
6	2.475	+0.003	2.483	+0.020	5.316	-0.054
7	2.472	0.000	2.462	-0.001	5.372	+0.002
10	2.510	+0.038	2.787	+0.314	4.654	-0.716
11	2.476	+0.004	2.488	+0.025	5.304	-0.066
12	2.472	—	2.463	—	5.370	—
20	2.488	+0.016	2.606	+0.143	4.988	-0.382
21	2.479	+0.007	2.521	+0.058	5.212	-0.158
22	2.473	+0.001	2.467	+0.004	5.361	-0.009
30	2.489	+0.017	2.619	+0.146	4.969	-0.401
31	2.477	+0.005	2.488	+0.025	5.311	-0.059
32	2.473	+0.001	2.470	+0.007	5.351	-0.019

The deviations refer to the differences between the values obtained with `failsafe = 12`. For comparison, the values and theoretical uncertainties calculated with `failsafe = 12` are:

$$Y_p = (2.472 \pm 0.003) \times 10^{-1}, \quad (\text{C.1})$$

$${}^2\text{H}/\text{H} = (2.463 \pm 0.038) \times 10^{-5}, \quad (\text{C.2})$$

$${}^7\text{Li}/\text{H} = (5.370 \pm 0.352) \times 10^{-10}. \quad (\text{C.3})$$

The computation times with different compilers (with `OpenMP` activated unless specified otherwise) on an Intel Core i7-6700HQ with 4 cores at 2.60GHz are for one BBN calculation:

failsafe	gcc 8.3	clang 7.0	icc 19.0	gcc w/o OpenMP
0	0.0292439 s	0.0371051 s	0.035774 s	0.030885 s
1	0.0598671 s	0.0702951 s	0.04901 s	0.0663319 s
2	0.12691 s	0.14824 s	0.105646 s	0.139108 s
3	0.513578 s	0.595697 s	0.426564 s	0.561526 s
5	0.241128 s	0.279304 s	0.201433 s	0.260442 s
6	1.83213 s	2.23646 s	1.5879 s	1.97117 s
7	16.9573 s	20.3785 s	14.0737 s	17.8445 s
10	0.843263 s	0.992097 s	0.688307 s	0.89699 s
11	7.81774 s	9.06975 s	6.40297 s	8.35519 s
12	72.1262 s	84.8538 s	59.8514 s	77.7767 s
20	0.157761 s	0.176582 s	0.133258 s	0.169685 s
21	0.54873 s	0.625423 s	0.449118 s	0.597708 s
22	4.84355 s	5.62732 s	4.11268 s	5.26304 s
30	0.200886 s	0.231516 s	0.170542 s	0.215176 s
31	1.53365 s	1.79631 s	1.29438 s	1.68841 s
32	5.85279 s	6.8085 s	4.89614 s	6.41176 s

For a single BBN calculation, `OpenMP` does not decrease the computation time. However `OpenMP` decreases the computation times if `REACLIB` is activated and the number of elements is larger.

The computation times of the correlation matrix are:

failsafe	gcc 8.3	clang 7.0	icc 19.0	gcc w/o OpenMP
0	0.710522 s	0.945136 s	0.618748 s	2.5015 s
1	1.98385 s	2.63015 s	1.73446 s	7.17126 s
2	4.37474 s	6.00581 s	3.70563 s	14.8925 s
3	17.5996 s	22.5279 s	14.915 s	59.1254 s
5	8.0979 s	9.99622 s	6.89687 s	27.2925 s
6	64.6826 s	77.3424 s	51.4998 s	207.582 s
7	549.041 s	721.656 s	467.687 s	1886.54 s
10	27.4793 s	34.2517 s	24.0487 s	94.5529 s
11	256.467 s	321.405 s	218.789 s	882.831 s
12	2344.66 s	3094.74 s	2062.58 s	8312.86 s
20	4.97176 s	7.24952 s	4.54886 s	17.6842 s
21	17.3305 s	28.8574 s	15.2907 s	62.2697 s
22	151.593 s	224.326 s	135.464 s	550.91 s
30	6.28095 s	8.24639 s	5.8179 s	22.2994 s
31	48.7308 s	67.6528 s	44.2791 s	176.329 s
32	188.842 s	258.642 s	175.523 s	671.334 s

D BBN constraints

The function `bbn_excluded` contains a recent set of uncorrelated observational measurements [28],

$$\begin{aligned} Y_p &= 0.2450 \pm 0.003, \\ [{}^2\text{H}]/[\text{H}] &= (2.569 \pm 0.027) \times 10^{-5}, \\ [{}^3\text{He}]/[{}^2\text{H}] &= (1.1 \pm 0.2) \times 10^{-5}, \\ [{}^7\text{Li}]/[\text{H}] &= (1.6 \pm 0.3) \times 10^{-10}. \end{aligned} \tag{D.1}$$

To assess the validity of the calculated abundances, a χ^2 is computed using

$$\chi^2 = \sum_i (O_i - E_i) C_{ij}^{-1} (O_j - E_j), \tag{D.2}$$

where i corresponds to the abundances used to set the constraints, O_i to the calculated abundance, E_i to the observational measurement and C_{ij}^{-1} to the inverse of the sum of the theoretical and experimental covariance matrices.

The number of degrees of freedom is considered to be the number of observational constraints, which can be set into `constraints`, and the exclusion is assessed at 95 % C.L. If `err` is smaller than 3, the correlations are ignored, otherwise the covariance matrix is computed.

By default, only the two constraints on Y_p and $[{}^2\text{H}]/[\text{H}]$ are used to compute the χ^2 .

The user is invited to change the constraints with up-to-date values in the routine `bbn_excluded`, which can be found in `src/bbn_constraints.c`.

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Table 2. Table summarizing the conventions used to denote the nuclear reactions. Reverse reaction rates are detailed balance factors. Energy release is given in K.

reaction index	reaction	reverse reaction rate	energy release
0	n decay	0	0
1	$n \rightarrow p$	0	0
2	${}^3\text{H} \rightarrow e^- + \bar{\nu}_e + {}^3\text{He}$	0	0
3	${}^8\text{Li} \rightarrow e^- + \bar{\nu}_e + 2{}^4\text{He}$	0	0
4	${}^{12}\text{B} \rightarrow e^- + \bar{\nu}_e + {}^{12}\text{C}$	0	0
5	${}^{14}\text{C} \rightarrow e^- + \bar{\nu}_e + {}^{14}\text{N}$	0	0
6	${}^8\text{B} \rightarrow e^+ + \nu_e + 2{}^4\text{He}$	0	0
7	${}^{11}\text{C} \rightarrow e^+ + \nu_e + {}^{11}\text{B}$	0	0
8	${}^{12}\text{N} \rightarrow e^+ + \nu_e + {}^{12}\text{C}$	0	0
9	${}^{13}\text{N} \rightarrow e^+ + \nu_e + {}^{13}\text{C}$	0	0
10	${}^{14}\text{O} \rightarrow e^+ + \nu_e + {}^{14}\text{N}$	0	0
11	${}^{15}\text{O} \rightarrow e^+ + \nu_e + {}^{15}\text{N}$	0	0
12	$\text{H} + n \rightarrow \text{p} + {}^2\text{H}$	0.477	25.815
13	${}^2\text{H} + n \rightarrow \gamma + {}^3\text{H}$	1.65	72.612
14	${}^3\text{He} + n \rightarrow \gamma + {}^4\text{He}$	2.63	238.794
15	${}^6\text{Li} + n \rightarrow \gamma + {}^7\text{Li}$	1.20	84.132
16	${}^3\text{He} + n \rightarrow \text{p} + {}^3\text{H}$	1.001	8.863
17	${}^7\text{Be} + n \rightarrow \text{p} + {}^7\text{Li}$	1.001	19.080
18	${}^6\text{Li} + n \rightarrow \alpha + {}^3\text{H}$	1.068	55.503
19	${}^7\text{Be} + n \rightarrow \alpha + {}^4\text{He}$	4.68	220.382
20	${}^2\text{H} + \text{p} \rightarrow \gamma + {}^3\text{He}$	1.65	63.749
21	${}^3\text{H} + \text{p} \rightarrow \gamma + {}^4\text{He}$	2.63	229.931
22	${}^6\text{Li} + \text{p} \rightarrow \gamma + {}^7\text{Be}$	1.20	65.053
23	${}^6\text{Li} + \text{p} \rightarrow \alpha + {}^3\text{He}$	1.067	46.640
24	${}^7\text{Li} + \text{p} \rightarrow \alpha + {}^4\text{He}$	4.68	201.302
25	${}^2\text{H} + \alpha \rightarrow \gamma + {}^6\text{Li}$	1.55	17.109
26	${}^3\text{H} + \alpha \rightarrow \gamma + {}^7\text{Li}$	1.13	28.629
27	${}^3\text{He} + \alpha \rightarrow \gamma + {}^7\text{Be}$	1.13	18.412
28	${}^2\text{H} + \text{d} \rightarrow \text{n} + {}^3\text{He}$	1.73	37.934
29	${}^2\text{H} + \text{d} \rightarrow \text{p} + {}^3\text{H}$	1.73	46.798
30	${}^3\text{H} + \text{d} \rightarrow \text{n} + {}^4\text{He}$	5.51	204.116
31	${}^3\text{He} + \text{d} \rightarrow \text{p} + {}^4\text{He}$	5.51	212.979
32	${}^3\text{He} + {}^3\text{He} \rightarrow 2\text{p} + {}^4\text{He}$	3.35	149.229
33	${}^7\text{Li} + \text{d} \rightarrow \text{n} + \alpha + {}^4\text{He}$	9.81	175.487
34	${}^7\text{Be} + \text{d} \rightarrow \text{p} + \alpha + {}^4\text{He}$	9.83	194.566
35	${}^3\text{He} + {}^3\text{H} \rightarrow \gamma + {}^6\text{Li}$	2.47	183.290
36	${}^6\text{Li} + \text{d} \rightarrow \text{n} + {}^7\text{Be}$	2.52	39.237
37	${}^6\text{Li} + \text{d} \rightarrow \text{p} + {}^7\text{Li}$	2.52	58.317
38	${}^3\text{He} + {}^3\text{H} \rightarrow \text{d} + {}^4\text{He}$	1.59	166.181
39	${}^3\text{H} + {}^3\text{H} \rightarrow 2\text{n} + {}^4\text{He}$	3.34	131.503
40	${}^3\text{He} + {}^3\text{H} \rightarrow \text{n} + \text{p} + {}^4\text{He}$	3.34	140.366

Table 3. Second part of table 2.

reaction index	reaction	reverse reaction rate	energy release
41	${}^7\text{Li} + {}^3\text{H} \rightarrow \text{n} + {}^9\text{Be}$	3.55	121.136
42	${}^7\text{Be} + {}^3\text{H} \rightarrow \text{p} + {}^9\text{Be}$	3.55	140.215
43	${}^7\text{Li} + {}^3\text{He} \rightarrow \text{p} + {}^9\text{Be}$	3.55	129.999
44	${}^7\text{Li} + \text{n} \rightarrow \gamma + {}^8\text{Li}$	1.33	23.589
45	${}^{10}\text{B} + \text{n} \rightarrow \gamma + {}^{11}\text{B}$	3.07	132.920
46	${}^{11}\text{B} + \text{n} \rightarrow \gamma + {}^{12}\text{B}$	2.37	39.111
47	${}^{11}\text{C} + \text{n} \rightarrow \text{p} + {}^{11}\text{B}$	1.001	32.086
48	${}^{10}\text{B} + \text{n} \rightarrow \alpha + {}^7\text{Li}$	0.755	32.371
49	${}^7\text{Be} + \text{p} \rightarrow \gamma + {}^8\text{B}$	1.32	1.595
50	${}^9\text{Be} + \text{p} \rightarrow \gamma + {}^{10}\text{B}$	0.986	76.424
51	${}^{10}\text{B} + \text{p} \rightarrow \gamma + {}^{11}\text{C}$	3.07	100.834
52	${}^{11}\text{B} + \text{p} \rightarrow \gamma + {}^{12}\text{C}$	7.10	185.173
53	${}^{11}\text{C} + \text{p} \rightarrow \gamma + {}^{12}\text{N}$	2.37	6.979
54	${}^{12}\text{B} + \text{p} \rightarrow \text{n} + {}^{12}\text{C}$	3.00	146.061
55	${}^9\text{Be} + \text{p} \rightarrow \alpha + {}^6\text{Li}$	0.618	24.663
56	${}^{10}\text{B} + \text{p} \rightarrow \alpha + {}^7\text{Be}$	0.754	13.291
57	${}^{12}\text{B} + \text{p} \rightarrow \alpha + {}^9\text{Be}$	0.291	79.903
58	${}^6\text{Li} + \alpha \rightarrow \gamma + {}^{10}\text{B}$	1.60	51.761
59	${}^7\text{Li} + \alpha \rightarrow \gamma + {}^{11}\text{B}$	4.07	100.549
60	${}^7\text{Be} + \alpha \rightarrow \gamma + {}^{11}\text{C}$	4.07	87.543
61	${}^8\text{B} + \alpha \rightarrow \text{p} + {}^{11}\text{C}$	3.07	85.948
62	${}^8\text{Li} + \alpha \rightarrow \text{n} + {}^{11}\text{B}$	3.07	76.960
63	${}^9\text{Be} + \alpha \rightarrow \text{n} + {}^{12}\text{C}$	10.28	66.158
64	${}^9\text{Be} + \text{d} \rightarrow \text{n} + {}^{10}\text{B}$	2.06	50.609
65	${}^{10}\text{B} + \text{d} \rightarrow \text{p} + {}^{11}\text{B}$	6.42	107.105
66	${}^{11}\text{B} + \text{d} \rightarrow \text{n} + {}^{12}\text{C}$	14.85	159.357
67	${}^4\text{He} + \alpha + \text{n} \rightarrow \gamma + {}^9\text{Be}$	0.600	18.262
68	${}^4\text{He} + 2\text{a} \rightarrow \gamma + {}^{12}\text{C}$	2.06	84.420
69	${}^8\text{Li} + \text{p} \rightarrow \text{n} + \alpha + {}^4\text{He}$	3.54	177.713
70	${}^8\text{B} + \text{n} \rightarrow \text{p} + \alpha + {}^4\text{He}$	3.55	218.787
71	${}^9\text{Be} + \text{p} \rightarrow \text{d} + \alpha + {}^4\text{He}$	0.796	7.554
72	${}^{11}\text{B} + \text{p} \rightarrow 2\text{a} + {}^4\text{He}$	3.45	100.753
73	${}^{11}\text{C} + \text{n} \rightarrow 2\text{a} + {}^4\text{He}$	3.46	132.838
74	${}^{12}\text{C} + \text{n} \rightarrow \gamma + {}^{13}\text{C}$	0.898	57.400
75	${}^{13}\text{C} + \text{n} \rightarrow \gamma + {}^{14}\text{C}$	3.62	94.884
76	${}^{14}\text{N} + \text{n} \rightarrow \gamma + {}^{15}\text{N}$	2.74	125.715
77	${}^{13}\text{N} + \text{n} \rightarrow \text{p} + {}^{13}\text{C}$	1.001	34.846
78	${}^{14}\text{N} + \text{n} \rightarrow \text{p} + {}^{14}\text{C}$	3.00	7.263
79	${}^{15}\text{O} + \text{n} \rightarrow \text{p} + {}^{15}\text{N}$	1.001	41.037
80	${}^{15}\text{O} + \text{n} \rightarrow \alpha + {}^{12}\text{C}$	0.707	98.659

Table 4. Third part of table 2.

reaction index	reaction	reverse reaction rate	energy release
81	$^{12}\text{C} + \text{p} \rightarrow \gamma + ^{13}\text{N}$	0.896	22.554
82	$^{13}\text{C} + \text{p} \rightarrow \gamma + ^{14}\text{N}$	1.21	87.621
83	$^{14}\text{C} + \text{p} \rightarrow \gamma + ^{15}\text{N}$	0.912	118.452
84	$^{13}\text{N} + \text{p} \rightarrow \gamma + ^{14}\text{O}$	3.62	53.705
85	$^{14}\text{N} + \text{p} \rightarrow \gamma + ^{15}\text{O}$	2.73	84.678
86	$^{15}\text{N} + \text{p} \rightarrow \gamma + ^{16}\text{O}$	3.67	140.733
87	$^{15}\text{N} + \text{p} \rightarrow \alpha + ^{12}\text{C}$	0.706	57.622
88	$^{12}\text{C} + \alpha \rightarrow \gamma + ^{16}\text{O}$	5.20	83.111
89	$^{10}\text{B} + \alpha \rightarrow \text{p} + ^{13}\text{C}$	9.35	47.134
90	$^{11}\text{B} + \alpha \rightarrow \text{p} + ^{14}\text{C}$	11.03	9.098
91	$^{11}\text{C} + \alpha \rightarrow \text{p} + ^{14}\text{N}$	3.68	33.921
92	$^{12}\text{N} + \alpha \rightarrow \text{p} + ^{15}\text{O}$	4.25	111.620
93	$^{13}\text{N} + \alpha \rightarrow \text{p} + ^{16}\text{O}$	5.80	60.557
94	$^{10}\text{B} + \alpha \rightarrow \text{n} + ^{13}\text{N}$	9.34	12.288
95	$^{11}\text{B} + \alpha \rightarrow \text{n} + ^{14}\text{N}$	3.67	1.835
96	$^{12}\text{B} + \alpha \rightarrow \text{n} + ^{15}\text{N}$	4.25	88.439
97	$^{13}\text{C} + \alpha \rightarrow \text{n} + ^{16}\text{O}$	5.79	25.711
98	$^{11}\text{B} + \text{d} \rightarrow \text{p} + ^{12}\text{B}$	4.96	13.296
99	$^{12}\text{C} + \text{d} \rightarrow \text{p} + ^{13}\text{C}$	1.88	31.585
100	$^{13}\text{C} + \text{d} \rightarrow \text{p} + ^{14}\text{C}$	7.58	69.069