AlterBBN v2: A public code for calculating Big-Bang nucleosynthesis constraints in alternative cosmologies

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URL: https://alterbbn.hepforge.org/

Abstract

We present the version 2 of `AlterBBN`, an open public code for the calculation of the abundance of the elements from Big-Bang nucleosynthesis. It does not rely on any closed external library or program, aims at being user-friendly and allowing easy modifications, and provides a fast and reliable calculation of the Big-Bang nucleosynthesis constraints in the standard and alternative cosmologies.

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Contents

1 Introduction

`AlterBBN` is a public C program for the calculation of the abundance of the elements generated during Big-Bang nucleosynthesis (BBN), released under the GPL version 3 licence. Its first version was released in 2011 [1] and could be considered as a spiritual successor of `NUC123` [2]. Contrary to other public BBN codes such as the Fortran program `PArthENoPE` [3] or the Mathematica code `PRIMAT` [4], `AlterBBN` does not rely on any closed library or program. The main purpose of `AlterBBN` is to provide a fast and reliable calculation of the abundance of the elements in the standard model of cosmology as well as in alternative scenarios. `AlterBBN` is also included in the `SuperIso Relic` package [5, 6].

`AlterBBN` can be downloaded from its new website:

$$\texttt{https://alterbbn.hepforge.org/}$$

and involves an enlarged development team.

In `AlterBBN v2`, an automatic calculation of errors and correlations has been implemented using methods similar to the ones described in Refs. [7, 8]. To improve the speed of the calculations, a parallel processing is possible through the `OpenMP` library. In addition, the units throughout the code have been unified to have GeV as the main unit, in order to get consistency with `SuperIso Relic`. The nuclear reaction network has also been extended, and the code has therefore been scrutinised and improved for precision and speed.

Moreover, new cosmological scenarios have been added, such as a scenario with reheating, or with a decaying primordial scalar field. In addition, scenarios with WIMPs have been implemented in `AlterBBN`.

The rest of this paper is organised as follows. Section 2 provides a review of BBN physics and the cosmological modifications implemented in `AlterBBN`. Section 3 describes the content of the `AlterBBN` package, and Section 4 gives usage instructions. Section 5 describes the input and output of `AlterBBN`, and Section 6 provides example of analyses which can be performed with `AlterBBN`. Short descriptions of the nuclear reaction network and the BBN constraints implemented in `AlterBBN` are given in the appendices.

2 BBN physics and cosmology

In this section we briefly present the physics relevant for BBN studies. We consider the system of natural units $c = \hbar = k_{\mathrm{B}} = 1$.

2.1 Cosmological standard model and BBN

2.1.1 General equations

At the beginning of the BBN epoch, the Universe contains photons γ, electrons e^- and positrons e^+, protons p and neutrons n, and neutrinos ν. During BBN, new nuclei will form over nuclear reactions (see Table 1 in Appendix A), which are contained in the general name of baryons b. BBN takes place in the more global frame of the early Universe expansion, parametrised by the expansion rate $\dot{a}$ given by the Friedmann equation as a function of the total density $\rho_{\rm tot}$:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm tot}\,, \tag{2.1}$$

where H is the Hubble parameter and G is the Newton gravitational constant. This is completed by the equation of energy conservation:

$$\frac{\rm d}{{\rm d}t}(\rho_{\rm tot}a^3) + P_{\rm tot}\frac{\rm d}{{\rm d}t}(a^3) - a^3\dot{\rho}_{\rm tot}\bigg|_{T={\rm const}} = 0\,, \tag{2.2}$$

where the adiabaticity condition ${\rm d}(sa^3)/{\rm d}t = 0$ on the entropy density s is considered as fulfilled.

In these equations, the total density $\rho_{\rm tot}$ and pressure $P_{\rm tot}$ are given by the sums over all the aforementioned constituents:

$$\rho_{\rm tot} = \rho_\gamma + \rho_\nu + \rho_{\rm b} + \rho_{e^-} + \rho_{e^+}\,, \tag{2.3}$$

$$P_{\rm tot} = P_\gamma + P_\nu + P_{\rm b} + P_{e^-} + P_{e^+}\,. \tag{2.4}$$

They can be computed through statistical mechanics to give for photons:

$$\rho_\gamma = \frac{\pi^2}{15}T^4\,, \quad P_\gamma = \frac{1}{3}\rho_\gamma\,, \tag{2.5}$$

and for neutrinos:

$$\rho_\nu = N_\nu\frac{7}{8}\frac{\pi^2}{15}T_\nu^4\,, \quad P_\nu = \frac{1}{3}\rho_\nu\,, \tag{2.6}$$

where N_ν is the number of Standard Model neutrino species, modified by the non-exact relativistic behaviour of $e^\pm$ to the value $N_\nu = 3.046$ [9]. The factor $T_\nu/T = (4/11)^{1/3}$ comes from the neutrino decoupling which happens before BBN.

On the other hand, we parametrise the sums of $e^\pm$ densities and pressures thanks to the modified Bessel functions K_i [10]:

$$\rho_{e^-} + \rho_{e^+} = \frac{2}{\pi^2}m_e^4\sum_{n=1}^{\infty}(-1)^{n+1}\cosh(n\phi_e)M(nz)\,, \tag{2.7}$$

$$P_{e^-} + P_{e^+} = \frac{2}{\pi^2}m_e^4\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{nz}\cosh(n\phi_e)L(nz)\,, \tag{2.8}$$

where we have defined the adimensioned electron mass $z = m_e/T$ and chemical potential $\phi_{e^-} = -\phi_{e^+} \equiv \phi_e = \mu_e/T$ and:

$$L(z) = \frac{K_2(z)}{z}, \quad M(z) = \frac{1}{z}\left(\frac{3}{4}K_3(z) + \frac{1}{4}K_1(z)\right). \tag{2.9}$$

For the purpose of our computation, these convergent sums will be truncated at $n = 7$. Another equation comes from the charge conservation of the Universe. The difference between the $e^\pm$ densities is linked to the nuclei abundances through:

$$n_{e^-} - n_{e^+} = \frac{h_\eta T^3 S}{M_\mathrm{u}}, \tag{2.10}$$

with:

$$S = \sum_i Z_i Y_i, \tag{2.11}$$

where Z_i and Y_i are the charge number and abundance of nucleus i respectively. The variable h parametrises the baryon-to-photon ratio in the following way [2, 11, 12]:

$$h_\eta(T) = M_\mathrm{u}\frac{n_\gamma(T)}{T^3}\eta(T), \tag{2.12}$$

where M_u is the unit atomic mass and η the baryon-to-photon ratio. The difference in Eq. (2.10) can also be parametrised as:

$$n_{e^-} - n_{e^+} = \frac{2}{\pi^2}m_e^3\sum_{i=1}^\infty (-1)^{n+1}\sinh(n\phi_e)L(nz). \tag{2.13}$$

Using Eqs. (2.10) and (2.13) the electron chemical potential can be determined through:

$$\frac{\mathrm{d}\phi_e}{\mathrm{d}t} = \frac{\partial\phi_e}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}t} + \frac{\partial\phi_e}{\partial a}\frac{\mathrm{d}a}{\mathrm{d}t} + \frac{\partial\phi_e}{\partial S}\frac{\mathrm{d}S}{\mathrm{d}t}. \tag{2.14}$$

Finally, the baryon density and pressure are given by the sums on the i nuclei [11]:

$$\rho_\mathrm{b} = h_\eta T^3\left(1 + \sum_i\left(\frac{\Delta M_i}{M_\mathrm{u}} + \zeta T\right)Y_i\right), \tag{2.15}$$

$$P_\mathrm{b} = h_\eta T^3\left(\frac{2}{3}\zeta T\sum_i Y_i\right), \tag{2.16}$$

where ΔM_i is the mass excess of nucleus i (see Table 1 in Appendix A) and $\zeta = 3/2M_\mathrm{u}$. The parameter h_η can be determined dynamically through $h_\eta \sim \rho_\mathrm{b}/T^3 \sim 1/a^3T^3$ which implies the logarithmic relation:

$$\frac{\mathrm{d}\ln(h_\eta)}{\mathrm{d}t} = -3\left(\frac{\mathrm{d}\ln(a)}{\mathrm{d}t} + \frac{\mathrm{d}\ln(T)}{\mathrm{d}t}\right). \tag{2.17}$$

2.1.2 Nuclear reactions

The set of nuclear reactions used in `AlterBBN` is given in Tables 2 and 3 in Appendix A. Each one of them can be written under the generalised form (to take into account reactions where 3 nuclei are involved [13]):

$$N_i \, ^{A_i}Z_i + N_j \, ^{A_j}Z_j + N_k \, ^{A_k}Z_k \longleftrightarrow N_l \, ^{A_l}Z_l + N_m \, ^{A_m}Z_m + N_n \, ^{A_n}Z_n \,, \qquad (2.18)$$

where N_i is the number of nuclei Z_i that enters into the reaction and A_i is their atomic number (see Table 1 in Appendix A). Then the abundance evolution of any nuclei i is given by the equation:

$$\frac{\mathrm{d}Y_i}{\mathrm{d}t} = \sum_{j,k,l,m,n} N_i \left(-\frac{Y_i^{N_i} Y_j^{N_j} Y_k^{N_k}}{N_i! N_j! N_k!} \Gamma_{ijk \to lmn} + \frac{Y_l^{N_l} Y_m^{N_m} Y_n^{N_n}}{N_l! N_m! N_n!} \Gamma_{lmn \to ijk} \right) \,, \qquad (2.19)$$

where $\Gamma_{ijk \to lmn}$ and $\Gamma_{lmn \to ijk}$ are the forward and reverse reaction rates respectively.

2.1.3 Initial conditions

The dynamical variables of interest are $h(t)$, $\phi_e(t)$ and $Y_i(t)$, all functions of time, or equivalently of temperature. The initial temperature is denoted by T_i. The initial condition for h depends on the initial value of the baryon-to-photon ratio η_i which is obtained from entropy conservation:

$$h_\eta(T_\mathrm{i}) = M_\mathrm{u} \frac{n_\gamma(T_\mathrm{i})}{T_\mathrm{i}^3} \eta_0 \left(1 + \frac{s_{e^\pm}(T_\mathrm{i})}{s_\gamma(T_\mathrm{i})} \right) \,, \qquad (2.20)$$

where η_0 is the CMB baryon-to-photon ratio and for any species the entropy density reads:

$$s_\mathrm{sp} = \frac{\rho_\mathrm{sp} + P_\mathrm{sp}}{T_\mathrm{sp}} \,. \qquad (2.21)$$

The initial condition for ϕ_e is:

$$\phi_e(T_\mathrm{i}) \approx \frac{\pi^2}{2} \frac{h_\eta(T_\mathrm{i}) Y_\mathrm{p}}{M_\mathrm{u} z_\mathrm{i}^3} \frac{1}{\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n L(n z_\mathrm{i})} \,, \qquad (2.22)$$

where Y_p is the initial proton abundance and z_i is the initial adimensioned electron mass. The initial proton and neutron abundances Y_p and Y_n are given by the equilibrium of the reaction p $\longleftrightarrow$ n:

$$Y_\mathrm{p}(T_\mathrm{i}) = \frac{1}{1 + e^{-q/T_\mathrm{i}}} \,, \qquad Y_\mathrm{n}(T_\mathrm{i}) = \frac{1}{1 + e^{q/T_\mathrm{i}}} \,, \qquad (2.23)$$

where $q = m_\mathrm{n} - m_\mathrm{p}$ is the nucleon mass difference. A similar equilibrium equation is applied to find the (small) initial deuterium abundance [12]. All the other nucleus abundances are 0 because they have not started to form yet. The initial time can be

found from the initial temperature through an "infinite temperature" approximation [12]:

$$t_{\mathrm{i}} = \frac{\sqrt{12\pi G\sigma}}{T_{\mathrm{i}}^2} \, , \tag{2.24}$$

where $\sigma = \pi^2/60$ is the Stefan-Boltzmann constant.

The set of Eqs. (2.1), (2.2), (2.14), (2.17) and (2.19) will be solved by a 4th-order Runge-Kutta integration.

2.2 Modified cosmologies

In this section, we present the different modified scenarios, compared to the standard cosmology BBN, that are implemented in the `AlterBBN` program.

2.2.1 Modified expansion rate

The addition of any new component in the early Universe, such as WIMPs, equivalent neutrinos or any kind of effective "dark density", has an impact on the Friedmann equation (2.1). Indeed, the total density ρ_{tot} from previous section received a "dark" contribution ρ_{D}:

$$\rho_{\mathrm{tot}} \rightarrow \rho_{\mathrm{tot}} + \rho_{\mathrm{D}} \, , \tag{2.25}$$

$$H^2 = \frac{8\pi G}{3} \left(\rho_{\mathrm{tot}} + \rho_{\mathrm{D}} \right) \, . \tag{2.26}$$

This modification affects the computation of the Hubble parameter, as shown by Eq. (2.26). For the case of a "dark density", the following parametrisation has been used [14]:

$$\rho_{\mathrm{D}}(T) = \kappa_\rho \, \rho_\gamma(T_0) \left(\frac{T}{T_0} \right)^{n_\rho} \, , \tag{2.27}$$

where $T_0 = 1$ MeV, chosen as a typical energy scale to fit the BBN initial conditions. Thus, κ_ρ is the ratio of the dark density to the photon density at this energy scale. n_ρ is the decrease exponent of this dark density (4 for radiation, 3 for matter, *etc.*). A temperature cut can be added below which this density is strictly 0.

The pressure P_{D} associated to ρ_{D} is calculated automatically from a combination of the conservation equation and the entropy density evolution:

$$P_{\mathrm{D}} = s \, \frac{\mathrm{d}T}{\mathrm{d}s} \left(\frac{\mathrm{d}\rho_{\mathrm{D}}}{\mathrm{d}T} - \rho_{\mathrm{D}} \frac{\mathrm{d}s}{\mathrm{d}T} \right) \, , \tag{2.28}$$

where s is the entropy density. The total radiation entropy s_{rad} is parametrised through the effective relativistic entropy degrees of freedom h_{eff} as:

$$s_{\mathrm{rad}}(T) = h_{\mathrm{eff}}(T) \frac{2\pi^2}{45} T^3 \, . \tag{2.29}$$

The $h_{\mathrm{eff}}(T)$ are tabulated in the directory `sgStar_heff`.

Another consequence of this modification is the computation of the initial time in the `AlterBBN` program, as one can see from Eq. (2.24). As long as the density is larger at a given time, the Hubble parameter is larger, and thus the time is smaller. For a fixed temperature, a correction has to be applied to the initial time t_i following:

$$t_{i,D} = \frac{H_{SBBN}}{H_D} t_i \, , \tag{2.30}$$

where H_{SBBN} is the standard Hubble rate given in Eq. (2.1) and H_D the modified one given by Eq. (2.26).

2.2.2 Additional neutrino species

The `AlterBBN` program leaves the possibility to modify the number of Standard Model neutrino species through the value of N_ν (see Eq. (2.6)). It is possible to add equivalent neutrinos to this number through a contribution ΔN_ν: $N_{tot} = N_\nu + \Delta N_\nu$.

In addition, there could be a neutrino degeneracy between the 3 neutrino species of the Standard Model, which leads to non-zero adimensioned chemical potentials $\xi_{\nu,1}$, $\xi_{\nu,2}$ and $\xi_{\nu,3}$, which are not necessarily equal. This will have two consequences.

The first one is a modification of Eq. (2.6) for the computation of the neutrino density. The exact statistical mechanics formula is, for each species $i = 1, 2, 3$:

$$\rho_{\nu_i,\bar{\nu}_i} = \frac{1}{2\pi^2} T_\nu^4 \int_0^{+\infty} dx \frac{x^3}{1 + \exp(x \mp \xi_{\nu,i})} \, , \tag{2.31}$$

which for small degeneracies ($\xi_{\nu,i} < 0.3$) is approximated by the expansion [2]:

$$\rho_{\nu,i} + \rho_{\bar{\nu},i} \approx \frac{\pi^2}{15} T_\nu^4 \left(\frac{7}{8} + \frac{15}{4\pi^2} \xi_{\nu,i}^2 + \frac{15}{8\pi^4} \xi_{\nu,i}^4 + \mathcal{O}(\xi_{\nu,i}^4) \right) \, , \tag{2.32}$$

and for high degeneracies ($\xi_{\nu,i} > 30$) by the expansion [2]:

$$\rho_{\nu,i} + \rho_{\bar{\nu},i} \approx \frac{1}{8\pi^2} (T_\nu \xi_{\nu,i})^4 \left(1 + \frac{2\pi^2}{\xi_{\nu,i}^2} + \mathcal{O}\left(\frac{1}{\xi_{\nu,i}^2} \right) \right) \, . \tag{2.33}$$

For intermediate degeneracies ($0.3 < \xi_{\nu,i} < 30$), Eq. (2.31) has to be integrated numerically.

The second consequence is a modification of the weak interaction equilibrium in the reaction p $\longleftrightarrow$ n. The initial abundances of protons and neutrons, given in Eq. (2.23), become:

$$Y_p(T_i) = \frac{1}{1 + e^{-q/T_i - \xi_{\nu_e}}} \, , \quad Y_n(T_i) = \frac{1}{1 + e^{q/T_i + \xi_{\nu_e}}} \, . \tag{2.34}$$

2.2.3 Modification of the entropy content

The early Universe content can also be modified by adding entropy components, which can derive from particle annihilation, or simply be any kind of "dark entropy" density. The energy conservation given in Eq. (2.2) becomes:

$$\frac{d}{dt}(\rho_{tot} a^3) + P_{tot} \frac{d}{dt}(a^3) - a^3 \dot{\rho}_{tot} \bigg|_{T=const} - T \frac{d}{dt}(sa^3) = 0 \, , \tag{2.35}$$

where s is the total entropy density, which is composed of radiation and dark entropies, denoted by s_{rad} and s_{D}, respectively. Two different cases can occur (simultaneously or separately):

1 – Dark entropy: the "dark entropy" s_{D} is not linked to any reheating process of radiation, and two different parametrisations are possible. We can first parametrise it through [15]:

$$s_{\mathrm{D}} = \kappa_s s_\gamma(T_0) \left(\frac{T}{T_0}\right)^{n_s} , \tag{2.36}$$

where the energy scale at which the dark entropy to photon entropy ratio κ_s is taken is also $T_0 = 1$ MeV, and n_s is the decrease exponent of this component. A temperature cut below which this density is strictly 0 can be added.

A second parametrisation of the "dark entropy" is possible through an additional entropy injection Σ_{D} [16]:

$$\Sigma_{\mathrm{D}}(T) = \kappa_\Sigma \Sigma_{\mathrm{rad}}^{\mathrm{eff}}(T_0) \left(\frac{T}{T_0}\right)^{n_\Sigma} , \tag{2.37}$$

where κ_Σ is ratio of the dark entropy injection to the radiation entropy density time-derivative $\Sigma_{\mathrm{rad}}^{\mathrm{eff}}(T) \equiv \left|\frac{\mathrm{d}s_{\mathrm{rad}}}{\mathrm{d}t}\right| = 3Hs_{\mathrm{rad}}$ at $T_0 = 1$ MeV and n_Σ is the decrease exponent of this production. The associated "dark entropy" density is thus the integral:

$$s_{\mathrm{D}}(T) = 3\sqrt{\frac{5}{4\pi^3 G}} h_{\mathrm{eff}}(T) T^3 \int_0^T \mathrm{d}T' \frac{\sqrt{g_*(T)}\Sigma_{\mathrm{D}}(T')}{h_{\mathrm{eff}}^2(T') T'^6 \sqrt{1 + \rho_{\mathrm{D}}(T)/\rho_{\mathrm{rad}}(T)}} , \tag{2.38}$$

where the values of $h_{\mathrm{eff}}(T)$ and $g_{\mathrm{eff}}(T)$ are tabulated.

2 – Reheating: the radiation entropy s_{rad} can receive contributions from a radiation entropy injection Σ_{rad} such as

$$\frac{\mathrm{d}s_{\mathrm{rad}}}{\mathrm{d}t} = -3Hs_{\mathrm{rad}} + \Sigma_{\mathrm{rad}} , \tag{2.39}$$

which will modify the relation between the temperature and the time, and result in a "reheating" of the primordial plasma and a local increase of the radiation entropy density. We can note that in absence of Σ_{rad} this relation is equivalent to $\mathrm{d}(s_{\mathrm{rad}}a^3)/\mathrm{d}t = 0$, so that the radiation entropy term vanishes in Eq. (2.35).

We use the following parametrisation:

$$\Sigma_{\mathrm{rad}}(T) = \kappa_{\Sigma_{\mathrm{r}}} \Sigma_{\mathrm{rad}}^{\mathrm{eff}}(T_0) \left(\frac{T}{T_0}\right)^{n_{\Sigma_{\mathrm{r}}}} , \tag{2.40}$$

where κ_{Σ_r} is ratio of the radiation entropy injection to the radiation entropy density time-derivative $\Sigma_{\mathrm{rad}}^{\mathrm{eff}}(T) \equiv \left|\frac{\mathrm{d}s_{\mathrm{rad}}}{\mathrm{d}t}\right| = 3Hs_{\mathrm{rad}}$ at $T_0 = 1$ MeV and n_{Σ_r} is the decrease exponent of this production.

2.2.4 Decaying scalar field

In this scenario, a pressureless primordial scalar field is decaying. Its density ρ_ϕ follows the Boltzmann equation:

$$\frac{\mathrm{d}\rho_\phi}{\mathrm{d}t} = -3H\rho_\phi - \Gamma_\phi\rho_\phi \,, \tag{2.41}$$

where Γ_ϕ is the decay width of the scalar field. The scalar field decay results in radiation entropy injection such as:

$$\frac{\mathrm{d}s_{\mathrm{rad}}}{\mathrm{d}t} = -3Hs_{\mathrm{rad}} + \frac{\Gamma_\phi\rho_\phi}{T} \,. \tag{2.42}$$

The decay width can be related to the reheating temperature T_{RH} through [17]:

$$\Gamma_\phi = \sqrt{\frac{4\pi^3 g_{\mathrm{eff}}(T_{\mathrm{RH}})}{45}} \frac{T_{\mathrm{RH}}^2}{M_{\mathrm{P}}} \,, \tag{2.43}$$

where g_{eff} is the effective relativistic energy degrees of freedom, which can be obtained from the tables contained in `sgStar_heff`, and M_{P} is the Planck mass.

This scenario requires two input parameters, the first one being $\tilde{\rho}_\phi$ the scalar field energy density proportion to the photon energy density at the initial temperature (~ 2.3 MeV) and the second one the reheating temperature T_{RH}.

2.2.5 WIMP scenarios

Different WIMP scenarios have been implemented in `AlterBBN` [13]. WIMPs are part of the candidates for the DM problem in cosmology. They would consist in light, weakly interacting new particles characterised by their mass m_χ, their type (Majorana or Dirac fermion, real or complex scalar) and their couplings to the SM (neutrinos, and possibly equivalent neutrinos, or EM interactions).

Most ingredients needed to take WIMPs into account in the `AlterBBN` program have already been mentioned in previous paragraphs. They will consist in a new "dark density" component, as well as "dark entropy" production through self-annihilation. The WIMP density and pressure are given similarly to Eqs. (2.7) and (2.8) by [10]:

$$\rho_\chi = g_\chi m_\chi^4 \sum_{n=1}^{\infty} (-1)^{\beta(n+1)} \cosh(n\phi_\chi) M(nz_\chi) \,, \tag{2.44}$$

$$P_\chi = g_\chi m_\chi^4 \sum_{n=1}^{\infty} \frac{(-1)^{\beta(n+1)}}{nz_\chi} \cosh(n\phi_\chi) L(nz_\chi) \,, \tag{2.45}$$

where g_χ is the internal number of degrees of freedom of the WIMPs (1 for a real scalar, 2 for a complex scalar, 2 for a Majorana fermion, 4 for a Dirac fermion), $z_\chi = m_\chi/T_\chi$ is their adimensioned mass, and $\phi_\chi = \mu_\chi/T_\chi$ their adimensioned chemical potential. The temperature of the WIMPs T_χ is different (T or T_ν) depending on

the SM couplings of the WIMP particles (EM or neutrinos). In the formula, $\beta = 0$ for bosonic WIMPs and $\beta = 1$ for fermionic WIMPs. Finally, the "cosh" function has to be replaced by an "exp" function in the case of self-conjugate particles (real scalars and Majorana fermions).

WIMPs also contribute to the entropy density of the early Universe and thus the initial condition for the $h_\eta(T_\mathrm{i})$ variable from Eq. (2.20) has to be modified:

$$h_\eta(T_\mathrm{i}) = M_\mathrm{u} \frac{n_\gamma(T_\mathrm{i})}{T_\mathrm{i}^3} \eta_0 \left(1 + \frac{s_{e^\pm}(T_\mathrm{i}) + s_\chi(T_\mathrm{i})}{s_\gamma(T_\mathrm{i})} \right) , \tag{2.46}$$

Finally, WIMPs may dynamically modify the neutrino temperature if they are coupled to them. For details on the way these modifications alter the differential equations of BBN, we refer the reader to Section 3.2 of Ref. [13].

3 Content of the AlterBBN package

The folder `alterbbn_v2.0/` contains the 8 main programs of AlterBBN:

- `stand_cosmo.c`,
- `alter_eta.c`,
- `alter_neutrinos.c`,
- `alter_etannutau.c`,
- `alter_standmod.c`,
- `alter_reheating.c`,
- `alter_phi.c`,
- `alter_wimps.c`,

together with a `README` file, a `Makefile` file and a folder `alterbbn_v2.0/src/`. The folder `alterbbn_v2.0/src/` contains the source files:

- `bbn.c`,
- `bbnrate.c`,
- `general.c`,
- `cosmodel.c`,

together with the files `include.h` – containing the headers of all the program routines – and `Makefile`. There is also a folder `alterBBN_v2.0/src/sgStar_heff/` containing numerically computed tables for $h_\mathrm{eff}(T)$ and $g_\mathrm{eff}(T)$, mentioned in Sections 2.2.1 and 2.2.3.

3.1 Parameter structure

There are two main parameter structures in the program `AlterBBN`, defined in the file `include.h`. The first one is:

```
typedef struct relicparam
/* structure containing the cosmological model parameters */
{
    int entropy_model,energy_model;
    double dd0,ndd,Tdend,Tddeq; // dark density
    double sd0,nsd,Tsend; // dark entropy
    double Sigmad0,nSigmad,TSigmadend; // dark entropy injection
    double Sigmarad0,nSigmarad,TSigmaradend; // standard entropy injection
    double nt0,nnt,Tnend;

    double quintn2,quintn3,quintn4,quintT12,quintT23,quintT34;

    int phi_model; // decaying scalar field model switch
    double eta_phi,Gamma_phi,rhot_phi_Tmax,rho_phi;
    double rhot_phi0,Tphi0;
    double T_RH;
    double Sigmatildestar;
    double Sigmatildestar_max;
    double Tstdstar_max;

    double mgravitino;

    double relicmass;
    int scalar;

    int solver;

    double T;
    double Y;
    double Tfo,Tmax;

    int full_comput;

    double table_eff[276][3];   // Reads values from the SgStar files

    int use_table_rhoPD;
    double table_rhoPD[2][NTABMAX];
```

```
    int size_table_rhoPD;


    /*--------------------*/
    /* AlterBBN parameters */
    /*--------------------*/

    int err;
    int failsafe;
    double eta0;                    // Initial Baryon to photon ratio
    double Nnu;                     // Number of Neutrinos (e+- included)
    double dNnu;                    // Number of extra neutrinos (delta N_nu)
    double life_neutron,life_neutron_error; // neutron lifetime
    double xinu1,xinu2,xinu3; // [e-,neutrino], [muon,neutrino], [tau,neutrino]
                                      respectively (degeneracy parameters)
    double m_chi;                   // Mass of WIMP
    double g_chi;
    double Tinit;
    int wimp;                       // Switch to enable (1) / disable (0) wimps
    int SMC_wimp;                   // wimp coupling to SM particles. 1 for EM,
                                       2 for neutrino, 3 for neut. and eq. neut.
    int selfConjugate;              // 1/0 for self-conjugate/non-self-conjugate WIMP
    int fermion;
    int EM_coupled, neut_coupled, neuteq_coupled;
    double chi2;
    int nobs;
}
relicparam;
```

and it contains all the parameters necessary to compute the BBN abundances of the elements, both in standard cosmology and in alternative cosmologies. This structure is common with SuperIso Relic and some of its parameters are not used in AlterBBN.

The second one is:

```
typedef struct errorparam
{
    int failsafe;
    int errnumber;
    double random[];
    double life_neutron;
}
```

```
errorparam;
```

and it contains the parameters needed to give the estimated errors linked to the computed abundances.

`AlterBBN` has different modes to compute the abundance of the elements, determined by the `failsafe` variable of the `relicparam` structure: 0 corresponds to a fast but less precise calculation, and 1 to 3 to more and more precise but slower calculations. In case of a very non-standard cosmological scenario, it is advisable to set `failsafe` to 1 or more. By default, the standard mode is the fast one.

3.2 Main routines

The main routines defined in the library `libbbn.a` – once compiled – are listed below:

- `void Init_cosmomodel(struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` initialises the `paramrelic` structure with SBBN values. It sets the number of neutrino species to `Nnu = 3.046` (including effects from non exactly relativistic $e^{\pm}$ [9]), the baryon-to-photon ratio to `eta0 = 6.09 × 10^{-10}` [18], the initial temperature to `Tinit = 27 × 10^9 K` (corresponding to 2.3 MeV, an adequate value before the real start of BBN) and the lifetime of the neutron `life_neutron = 880.2 s` [18] (with its associated error to `life_time_error = 1.0 s`). All the other parameters are set to 0.

- `void Init_cosmomodel_param(double eta, double Nnu, double dNnu, double life_neutron, double life_neutron_error, double xinu1, double xinu2, double xinu3, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies some parameters of the `paramrelic` structure with potentially non-standard values: the baryon-to-photon ratio `eta0`, the number of Standard Model neutrino species `Nnu`, the number of additional neutrino species `dNnu`, the eventual degeneracy of the Standard Model neutrinos `xinu1`, `xinu2` and `xinu3` (see Section 2.2.2) and finally the neutron lifetime `life_neutron` and the associated error `life_neutron_error`.

- `void Init_dark_density(double dd0, double ndd, double T_end, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark density described in Section 2.2.1. Here $\kappa_\rho =$ `dd0`, $n_\rho =$ `ndd` and `T_end` is the temperature cutoff at which the effective dark density is set to 0.

- `void Init_dark_entropy(double sd0, double nsd, double T_end, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark entropy density, in the case of no reheating, described in Section 2.2.3. Here $\kappa_s =$ `sd0`, $n_s =$ `nsd` and `T_end` is the temperature cutoff at which the effective dark entropy is set to 0.

- `void Init_dark_entropySigmaD(double Sigmad0, double nSigmad, double T_end, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the effective dark entropy production in the no-reheating case described in Section 2.2.3. Here $\kappa_\Sigma =$ `Sigmad0`, $n_\Sigma =$ `nSigmad` and `T_end` is the temperature cutoff at which the effective entropy production is set to 0.

- `void Init_entropySigmarad(double Sigmarad0, double nSigmarad, double T_end, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the radiation entropy production in the reheating case, described in Section 2.2.3. Here $\kappa_{\Sigma_r} =$ `Sigmarad0`, $n_{\Sigma_r} =$ `nSigmarad` and `T_end` is the cutoff temperature at which the radiation entropy production is set to 0.

- `void Init_scalarfield(double rhotilde_phi, double T_RH, double eta_phi, struct relicparam* paramrelic)`

  This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the decay of a scalar field during BBN, as described in Section 2.2.4. Here `rhotilde_phi` is the ratio of the scalar field density over the photon density at the initial temperature, and `T_RH` is the reheating temperature. The parameter `eta_phi` has no effect in `AlterBBN` and is set to 0.

- `void Init_wimp(double mass_wimp, int EM_coupled, int neut_coupled, int neuteq_coupled, int fermion, int`

```
selfConjugate, double g_chi, struct relicparam* paramrelic)
```

This routine defined in `cosmodel.c` specifies the parameters of the `paramrelic` structure related to the existence of WIMPs during BBN, as described in Section 2.2.5. Here the parameters related to WIMP injection are specified: the WIMP mass $m_\chi = $ `mass_wimp`, the SM couplings `EM_coupled`, `neut_coupled` and `neuteq_coupled` (all switches between 0/1 for inactive/active), and the type of WIMP particle `fermion` and `selfConjugate` (both switches between 0/1 which represent the 4 types of wimps described in paragraph 2.2.5).

- ```
  void rate_weak(double f[], struct relicparam* paramrelic,
  struct errorparam* paramerror)
  ```

  This routine defined in `bbnrate.c` computes the forward reaction rates of the β-decays corresponding to the processes (2–11) given in Table 2 in Appendix A and stores them into the (2–11) slots of the variable `f[]`. There is no reverse reaction so the slots (2–11) of the variable `r[]` do not need to be computed.

- ```
  void rate_pn(double f[], double r[], double T9, double Tnu, struct
  relicparam* paramrelic, struct errorparam*
  paramerror)
  ```

  This routine defined in `bbnrate.c` computes the forward and reverse reaction rates of the nuclear reaction 1 (proton-neutron conversion) given in Table 2 in Appendix A and stores them into the slot (1) of the variables `f[]` and `r[]`.

- ```
  void rate_all(double f[], double T9, struct relicparam*
  paramrelic, struct errorparam* paramerror)
  ```

  This routine defined in `bbnrate.c` computes the forward reaction rates of the nuclear reactions (12-100) given in Tables 2 and 3 in Appendix A and stores them into the (12-100) slots of the variable `f[]` (the reverse reaction rates will be estimated elsewhere through detailed balance factors contained in the variable `reacparam[][]` and stored in the variable `r[]`).

- ```
  int nucl(struct relicparam* paramrelic, double ratioH[])
  ```

  This routine defined in `bbn.c` is the main routine of the program, as it is the one that computes the BBN abundance ratios of all nuclei given in Table 1 in Appendix A and in particular the light elements `Yp` $= \rho(^4\text{He})/\rho_\text{b}$, `H2_H` $= [^2\text{H}]/[\text{H}]$, `He3_H` $= [^3\text{He}]/[\text{H}]$, `Li7_H` $= [^7\text{Li}]/[\text{H}]$, `Li6_H` $= [^6\text{Li}]/[\text{H}]$ and `Be7_H` $= [^7\text{Be}]/[\text{H}]$ (note that `He3_H` and `Li7_H` contain the contributions of post-BBN

decays of respectively `H3_H` and `Be7_H`). It returns 0 if the computation succeeded or 1 otherwise.

- `int bbn_excluded(struct relicparam* paramrelic)`

  This routine defined in `bbn.c` is a "container" function that calls the `nucl` routine and compares its results with BBN observational constrains summarised in Appendix B.1. It returns 0 if the constrains are satisfied, 1 if the abundances are not compatible with the observations and -1 if the computation fails.

- `int bbn_excluded_chi2(struct relicparam* paramrelic)`

  This routine defined in `bbn.c` is a "container" function that calls the `nucl` routine and performs a χ^2 analysis of the ^{2}H and Y_{p} values using their observational values and uncertainties given in Appendix B.2. It returns 0 if the confidence level of $95\,\%$ is satisfied, 1 if the abundances are not compatible with the observations and -1 if the computation fails.

3.3 Error & correlations

`AlterBBN` includes the error estimation of the computations of the BBN abundances of the nuclei. This estimation relies on the parameters `err` and `life_neutron_error` contained in the `relicparam` structure and the parameters `errnumber` and `random[]` in the `errorparam` structure (see Section 3.1), as well as estimated errors on the nuclear reaction rates.

The `err` parameter switches between five methods of evaluation of the abundances of the elements and their errors:

- `err` $= 0$: central values of the nuclear reaction rates are used for all reactions,

- `err` $= 1$: higher values are used for all reactions,

- `err` $= 2$: lower values are used for all reactions,

- `err` $= 3$: the covariance matrix is calculated through the variation of the parameters, using the higher value of the reaction rates and following the method of Ref. [8],

- `err` $= 4$: randomly Gaussian distributed values (between lower and higher) are used for all reactions.

The 8 main programs listed in the next section run successively the `err` $= 2$, `err` $= 0$ and `err` $= 1$ types in order to give associated "lower", central and "upper" values of the abundances. Then they run the `err` $= 3$ type for all the reactions in

order to compute a correlation matrix between the abundances, stored in the variable `corr_ratioH[][]`.

An additional run of the `err = 4` type is implemented to perform a Monte Carlo correlation analysis, but it is commented due to the extensive computational time (the abundance computation has to be done a lot of times, a number defined by the variable `niter = 1000`, by default).

The `err = 0` type is used in order to compare the computed abundances to the observed ones (conservative limits). The `err = 3` type is used to do a χ^2 analysis (observational uncertainties).

4 Compilation and installation instructions

`AlterBBN` has been written in C respecting the `C99` standard, and it has been tested with the GNU C and the Intel C compilers on Linux, Windows and Mac. The package can be downloaded at the address:

$$\text{https://alterbbn.hepforge.org/}$$

The package should be unpacked in the desired directory, creating the main directory

$$\text{alterbbn\_vX.X/}$$

containing the material described in section 3. If needed, the user's C compiler information and flags can be specified in the file `Makefile` in this main directory. In particular, some of the computations are made in parallel using the `OpenMP` library. The user should comment the corresponding lines if this library is not installed. More information is provided in the `README` file.

To compile the library `libbbn.a`, type `make` in the main folder. The library file will be created in the subfolder `src/`. To compile a specific program, type `make name` or `make name.c` in the main folder, where `name` can be:

- `stand_cosmo` (see Section 5.1),

- `alter_eta` (see Section 5.2.1),

- `alter_neutrinos` (see Section 5.2.3),

- `alter_etannutau` (see Section 5.2.2),

- `alter_standmod` (see Section 5.3),

- `alter_reheating` (see Section 5.4),

- `alter_phi` (see Section 5.5),

- `alter_wimps` (see Section 5.6).

5 Input and output description

In this section we give input and output instructions for the 8 programs listed in section 4.

5.1 Standard cosmology

The program `stand_cosmo.x` computes the BBN abundances of the nuclei as well as the associated errors and correlation matrix in the standard cosmological model. It takes one integer as argument, if it is 1 a precise and slow calculation will be performed, if it is 0 a fast but less precise calculation is done. The values of the baryon-to-photon ratio, the neutron lifetime and the number of neutrinos species are those fixed by the `Init_cosmomodel` routine. Running the program with:

`./stand_cosmo.x 1`

returns:

```
              Yp       H2/H      He3/H       Li7/H       Li6/H       Be7/H
  low:  2.472e-01  2.753e-05  3.098e-05  1.324e-09  3.479e-15  9.885e-10
 cent:  2.472e-01  2.467e-05  1.031e-05  5.372e-10  1.086e-14  5.083e-10
 high:  2.471e-01  2.401e-05  4.437e-06  2.206e-10  3.478e-14  2.094e-10

----------

With uncertainties:
              Yp       H2/H      He3/H       Li7/H       Li6/H       Be7/H
 value:  2.472e-01  2.467e-05  1.031e-05  5.372e-10  1.086e-14  5.083e-10
  +/- :  3.249e-04  6.900e-07  4.350e-06  2.390e-10  1.085e-14  2.304e-10

Correlation matrix:
              Yp       H2/H      He3/H       Li7/H       Li6/H       Be7/H
    Yp   1.000000   0.021985  -0.005382  -0.002037  -0.014079  -0.002377
  H2/H   0.021985   1.000000   0.198843   0.158750   0.040139   0.150177
 He3/H  -0.005382   0.198843   1.000000   0.988316   0.014065   0.988002
 Li7/H  -0.002037   0.158750   0.988316   1.000000   0.002789   0.999477
 Li6/H  -0.014079   0.040139   0.014065   0.002789   1.000000   0.002409
 Be7/H  -0.002377   0.150177   0.988002   0.999477   0.002409   1.000000
```

`Compatible with BBN constraints (conservative limits)`
`Compatible with BBN constraints (chi2 including correlations)`

5.2 Standard cosmology with modified parameters

5.2.1 Modification of the baryon-to-photon ratio

The program `alter_eta.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified value of the baryon-to-photon ratio η_0, taken as an input argument. Running the program with:

`./alter_eta.x 3e-10`

returns (hereafter, only the relevant part of the output which changes will be given):

```
                Yp        H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  2.385e-01  7.234e-05  1.586e-05  1.367e-10  3.002e-14  7.659e-11
  +/- :  3.187e-04  3.741e-06  5.535e-06  4.766e-11  2.973e-14  3.038e-11
```

Excluded by BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)

5.2.2 Modifications of the baryon-to-photon ratio, neutrino number and neutron lifetime

The program `alter_etannutau.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified value of the baryon-to-photon ratio, number of Standard Model neutrino species, number of additional neutrino species, neutron lifetime and possibly neutron lifetime error in s, all taken as input arguments in the aforementioned order. Running the program with:

`./alter_etannutau 6.09e-10 3.05 0.1 880.5 2.`

returns:

```
                Yp        H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  2.476e-01  2.383e-05  1.029e-05  5.610e-10  1.043e-14  5.333e-10
  +/- :  4.774e-04  9.739e-07  4.448e-06  2.476e-10  1.036e-14  2.395e-10
```

Compatible with BBN constraints (conservative limits)
Compatible with BBN constraints (chi2 including correlations)

5.2.3 Modifications of the neutrino number and degeneracies

The program `alter_neutrinos.x` computes the BBN abundances, errors and correlations in the standard cosmological model, but with a modified number of Standard Model neutrino species, number of additional neutrino species and possibly neutrino degeneracies $\xi_{\nu,i}$ (see Section 2.2.2), taken as input arguments in the aforementioned order. Running the program with:

`./alter_neutrinos.x 3.046 0.1 0.1 0.1 0.1`

returns:

```
                Yp        H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  2.248e-01  2.285e-05  1.000e-05  5.270e-10  8.829e-15  5.034e-10
  +/- :  2.992e-04  8.764e-07  4.254e-06  2.392e-10  8.787e-15  2.322e-10
```

Compatible with BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)

5.3 Modified expansion rate and entropy content

The program `alter_standmod.x` computes the BBN abundances, errors and correlations in a cosmology scenario without reheating where the expansion rate and the entropy content are modified by the injection of a dark component throughout the

BBN epoch (see Section 2.2.3). It takes 4 to 6 input arguments ordered as κ_ρ, n_ρ, κ_s and n_s, and possibly the cutoff temperatures in MeV for dark energy and dark entropy. Running the program with:

`./alter_standmod.x 0.1 3 0.1 4 0.  0.`

returns:

```
                Yp      H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  2.634e-01  4.222e-05  1.186e-05  3.316e-10  1.972e-14  2.835e-10
  +/- :  3.255e-04  1.593e-06  4.806e-06  1.334e-10  1.963e-14  1.200e-10
```

```
Excluded by BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)
```

5.4   Effective reheating scenario

The program `alter_reheating.x` computes the BBN abundances, errors and correlations in a cosmology scenario with reheating where dark energy and entropy are injected throughout the BBN epoch. It takes 5 to 9 input arguments ordered as κ_ρ, n_ρ, κ_{Σ_r}, n_{Σ_r}, the temperature cutoff for the injection in MeV and possibly κ_s, n_s, κ_Σ and n_Σ (see Section 2.2.3). Running the program with:

`./alter_reheating 0 0 1 6 0.01`

returns:

```
                Yp      H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  1.631e-01  2.756e-04  2.888e-05  1.682e-10  7.292e-14  2.842e-12
  +/- :  2.741e-04  1.200e-05  5.344e-06  8.598e-11  7.045e-14  6.113e-13
```

```
Excluded by BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)
```

5.5   Decaying scalar field scenario

The program `alter_phi.x` computes the BBN abundances, errors and correlations in a cosmological scenario where a scalar field is decaying throughout the BBN epoch. It takes two input arguments which are the scalar field density ratio $\tilde{\rho}_\phi$ and the reheating temperature T_{RH} in MeV (see Section 2.2.4). Running the program with:

`./alter_phi.x 0.1 10`

returns:

```
                Yp      H2/H      He3/H      Li7/H      Li6/H      Be7/H
 value:  2.469e-01  2.329e-05  1.018e-05  5.746e-10  1.016e-14  5.475e-10
  +/- :  9.177e-03  3.632e-06  4.400e-06  2.777e-10  1.026e-14  2.717e-10
```

```
Compatible with BBN constraints (conservative limits)
Compatible with BBN constraints (chi2 including correlations)
```

5.6   WIMP scenario

The program `alter_wimps.x` computes the BBN abundances, errors and correlations in a cosmological scenario where WIMPs are added to the Standard Model particles.
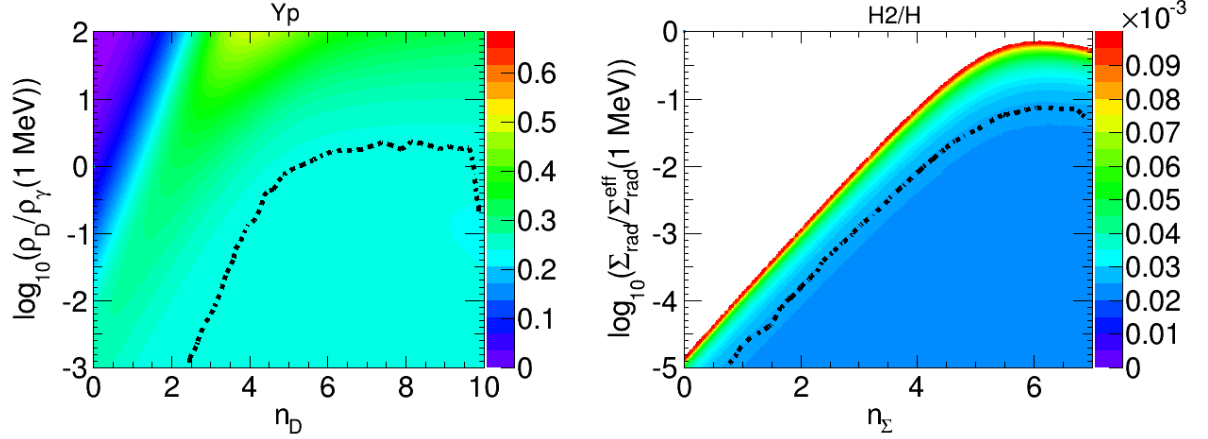
Figure 1. Y_p in the dark density scenario (left) and $[^2H]/[H]$ in the reheating scenario (right). The parameter regions excluded by BBN constraints are located above the black lines. The colours correspond to the values of Y_p and $[^2H]/[H]$.

It takes 3 input arguments, which are the type of WIMP particle (1: real scalar, 2: complex scalar, 3: Majorana fermion, 4: Dirac fermion), the type of coupling to the Standard Model particles (1: neutrinos, 2: EM, 3: equivalent neutrinos) and m_χ (to be given in MeV) (see Section 2.2.5). Running the program with:

```
./alter_wimp.x 2 2 15.
```

returns:

```
              Yp        H2/H       He3/H       Li7/H       Li6/H       Be7/H
 value:  2.489e-01  1.422e-05  8.801e-06  9.800e-10  6.304e-15  9.619e-10
  +/- :  7.574e-14  1.176e-17  4.150e-18  7.752e-23  1.082e-26  3.966e-23

Compatible with BBN constraints (conservative limits)
Excluded by BBN constraints (chi2 including correlations)
```

6 Example of results

To illustrate the capabilities of `AlterBBN`, we investigate the BBN constraints on the dark density and reheating scenarios, described in Sections 2.2.1 and 5.4, respectively. For the first case, we can constrain value of the the dark density at $T = 1$ MeV and its exponent, and for the second case, we can constrain the value of the radiation injection at $T = 1$ MeV and its exponent. We used `failsafe=3` for an improved precision. The results are shown in Fig. 1. The colour scale shows the value of Y_p in the dark density scenario, and of $[^2H]/[H]$ in the reheating scenario. The black lines are the limits obtained using the constraints described in Appendix B.2. The excluded region is above the lines.

A   Nuclear reaction network

Table 1. Table summarising the conventions used to denote the nuclei. Mass excess is given in MeV.

index	nuclei	atomic number	charge number	mass excess
1	n	1	0	8.071388
2	p	1	1	7.289028
3	^{2}H	2	1	13.135825
4	^{3}H	3	1	14.949915
5	^{3}He	3	2	14.931325
6	^{4}He	4	2	2.424931
7	^{6}Li	6	3	14.0864
8	^{7}Li	7	3	14.9078
9	^{7}Be	7	4	15.7696
10	^{8}Li	8	3	20.9464
11	^{8}B	8	5	22.9212
12	^{9}Be	9	4	11.34758
13	^{10}B	10	5	12.05086
14	^{11}B	11	5	8.6680
15	^{11}C	11	6	10.6506
16	^{12}B	12	5	13.3690
17	^{12}C	12	6	0
18	^{12}N	12	7	17.3382
19	^{13}C	13	6	3.125036
20	^{13}N	13	7	5.3455
21	^{14}C	14	6	3.019916
22	^{14}N	14	7	2.863440
23	^{14}O	14	8	8.006521
24	^{15}N	15	7	0.101439
25	^{15}O	15	8	2.8554
26	^{16}O	16	8	-4.737036

B BBN constraints

B.1 Conservative set

A conservative set of constraints from Ref. [19] is defined in the function `bbn_excluded`:

$$
\begin{aligned}
0.240 < Y_{\mathrm{p}} < 0.258 \,, \qquad & 1.2 \times 10^{-5} < [^2H]/[H] < 5.3 \times 10^{-5} \,, \\
0.57 < [^3H]/[^2H] < 1.52 \,, \qquad & [^7Li]/[H] > 0.85 \times 10^{-10} \,, \\
[^6Li]/[^7Li] < 0.66 \,, &
\end{aligned}
\tag{B.1}
$$

which constrains the helium abundance Y_{p} and the primordial $[^2H]/[H]$, $[^3H]/[^2H]$, $[^7Li]/[H]$ and $[^6Li]/[^7Li]$ ratios.

The user can change these constraints in the routine `bbn_excluded` which can be found in `src/bbn.c`.

B.2 χ^2 combination of recent constraints

The function `bbn_excluded_chi2` contains another recent set of uncorrelated observational measurements [18]:

$$
\begin{aligned}
Y_{\mathrm{p}} &= 0.2449 \pm 0.0040 \,, \\
[^2H]/[H] &= (2.527 \pm 0.030) \times 10^{-5} \,, \\
[^3H]/[^2H] &= (1.1 \pm 0.2) \times 10^{-5} \,, \\
[^7Li]/[H] &= (1.6 \pm 0.3) \times 10^{-10} \,.
\end{aligned}
\tag{B.2}
$$

To assess the validity of the calculated abundances, a χ^2 is computed using

$$
\chi^2 = \sum_i \left(O_i - E_i \right) C_{ij}^{-1} \left(O_j - E_j \right) ,
\tag{B.3}
$$

where i corresponds to the abundances used to set the constraints, O_i to the calculated abundance, E_i to the observational measurement and C_{ij}^{-1} to the inverse of the sum of the theoretical and experimental covariance matrices.

The number of degrees of freedom is considered to be the number of observational constraints, and the exclusion is assessed at 95 % C.L.

For the time-being, only the constraints of Y_{p} and $[^2H]/[H]$ are used to compute the χ^2. The user can change this in routine `bbn_excluded_chi2` which can be found in `src/bbn.c`.

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Table 2. Table summarising the conventions used to denote the nuclear reactions. Reverse reaction rates are detailed balance factors. Energy release is given in K.

reaction index	reaction	reverse reaction rate	energy release
0	n decay	0	0
1	n $\rightarrow$ p	0	0
2	^{3}H $\rightarrow$ e$^-$ + $\bar{\nu}_e$ + ^{3}He	0	0
3	^{8}Li $\rightarrow$ e$^-$ + $\bar{\nu}_e$ + 2^{4}He	0	0
4	^{12}B $\rightarrow$ e$^-$ + $\bar{\nu}_e$ + ^{12}C	0	0
5	^{14}C $\rightarrow$ e$^-$ + $\bar{\nu}_e$ + ^{14}N	0	0
6	^{8}B $\rightarrow$ e$^+$ + ν_e + 2^{4}He	0	0
7	^{11}C $\rightarrow$ e$^+$ + ν_e + ^{11}B	0	0
8	^{12}N $\rightarrow$ e$^+$ + ν_e + ^{12}C	0	0
9	^{13}N $\rightarrow$ e$^+$ + ν_e + ^{13}C	0	0
10	^{14}O $\rightarrow$ e$^+$ + ν_e + ^{14}N	0	0
11	^{15}O $\rightarrow$ e$^+$ + ν_e + ^{15}N	0	0
12	H + n $\rightarrow$ + ^{2}H	0.477	25.815
13	^{2}H + n $\rightarrow$ γ + ^{3}H	1.65	72.612
14	^{3}He + n $\rightarrow$ γ + ^{4}He	2.63	238.794
15	^{6}Li + n $\rightarrow$ γ + ^{7}Li	1.20	84.132
16	^{3}He + n $\rightarrow$ p + ^{3}H	1.001	8.863
17	^{7}Be + n $\rightarrow$ p + ^{7}Li	1.001	19.080
18	^{6}Li + n $\rightarrow$ α + ^{3}H	1.068	55.503
19	^{7}Be + n $\rightarrow$ α + ^{4}He	4.68	220.382
20	^{2}H + p $\rightarrow$ γ + ^{3}He	1.65	63.749
21	^{3}H + p $\rightarrow$ γ + ^{4}He	2.63	229.931
22	^{6}Li + p $\rightarrow$ γ + ^{7}Be	1.20	65.053
23	^{6}Li + p $\rightarrow$ α + ^{3}He	1.067	46.640
24	^{7}Li + p $\rightarrow$ α + ^{4}He	4.68	201.302
25	^{2}H + α $\rightarrow$ γ + ^{6}Li	1.55	17.109
26	^{3}H + α $\rightarrow$ γ + ^{7}Li	1.13	28.629
27	^{3}He + α $\rightarrow$ γ + ^{7}Be	1.13	18.412
28	^{2}H + d $\rightarrow$ n + ^{3}He	1.73	37.934
29	^{2}H + d $\rightarrow$ p + ^{3}H	1.73	46.798
30	^{3}H + d $\rightarrow$ n + ^{4}He	5.51	204.116
31	^{3}He + d $\rightarrow$ p + ^{4}He	5.51	212.979
32	^{3}He + ^{3}He $\rightarrow$ 2p + ^{4}He	3.35	149.229
33	^{7}Li + d $\rightarrow$ n + α + ^{4}He	9.81	175.487
34	^{7}Be + d $\rightarrow$ p + α + ^{4}He	9.83	194.566
35	^{3}He + ^{3}H $\rightarrow$ γ + ^{6}Li	2.47	183.290
36	^{6}Li + d $\rightarrow$ n + ^{7}Be	2.52	39.237
37	^{6}Li + d $\rightarrow$ p + ^{7}Li	2.52	58.317
38	^{3}He + ^{3}H $\rightarrow$ d + ^{4}He	1.59	166.181
39	^{3}H + ^{3}H $\rightarrow$ 2n + ^{4}He	3.34	131.503
40	^{3}He + ^{3}H $\rightarrow$ n + p + ^{4}He	3.34	140.366
41	^{7}Li + ^{3}H $\rightarrow$ n + ^{9}Be	3.55	121.136
42	^{7}Be + ^{3}H $\rightarrow$ p + ^{9}Be	3.55	140.215
43	^{7}Li + ^{3}He $\rightarrow$ p + ^{9}Be	3.55	129.999
44	^{7}Li + n $\rightarrow$ γ + ^{8}Li	1.33	23.589
45	^{10}B + n $\rightarrow$ γ + ^{11}B	3.07	132.920
46	^{11}B + n $\rightarrow$ γ + ^{12}B	2.37	39.111
47	^{11}C + n $\rightarrow$ p + ^{11}B	1.001	32.086
48	^{10}B + n $\rightarrow$ α + ^{7}Li	0.755	32.371
49	^{7}Be + p $\rightarrow$ γ + ^{8}B	1.32	1.595

Table 3. Second half of the table 2.

reaction index	reaction	reverse reaction rate	energy release
50	$^9\text{Be} + \text{p} \rightarrow \gamma + {}^{10}\text{B}$	0.986	76.424
51	$^{10}\text{B} + \text{p} \rightarrow \gamma + {}^{11}\text{C}$	3.07	100.834
52	$^{11}\text{B} + \text{p} \rightarrow \gamma + {}^{12}\text{C}$	7.10	185.173
53	$^{11}\text{C} + \text{p} \rightarrow \gamma + {}^{12}\text{N}$	2.37	6.979
54	$^{12}\text{B} + \text{p} \rightarrow \text{n} + {}^{12}\text{C}$	3.00	146.061
55	$^9\text{Be} + \text{p} \rightarrow \alpha + {}^6\text{Li}$	0.618	24.663
56	$^{10}\text{B} + \text{p} \rightarrow \alpha + {}^7\text{Be}$	0.754	13.291
57	$^{12}\text{B} + \text{p} \rightarrow \alpha + {}^9\text{Be}$	0.291	79.903
58	$^6\text{Li} + \alpha \rightarrow \gamma + {}^{10}\text{B}$	1.60	51.761
59	$^7\text{Li} + \alpha \rightarrow \gamma + {}^{11}\text{B}$	4.07	100.549
60	$^7\text{Be} + \alpha \rightarrow \gamma + {}^{11}\text{C}$	4.07	87.543
61	$^8\text{B} + \alpha \rightarrow \text{p} + {}^{11}\text{C}$	3.07	85.948
62	$^8\text{Li} + \alpha \rightarrow \text{n} + {}^{11}\text{B}$	3.07	76.960
63	$^9\text{Be} + \alpha \rightarrow \text{n} + {}^{12}\text{C}$	10.28	66.158
64	$^9\text{Be} + \text{d} \rightarrow \text{n} + {}^{10}\text{B}$	2.06	50.609
65	$^{10}\text{B} + \text{d} \rightarrow \text{p} + {}^{11}\text{B}$	6.42	107.105
66	$^{11}\text{B} + \text{d} \rightarrow \text{n} + {}^{12}\text{C}$	14.85	159.357
67	$^4\text{He} + \alpha + \text{n} \rightarrow \gamma + {}^9\text{Be}$	0.600	18.262
68	$^4\text{He} + 2a \rightarrow \gamma + {}^{12}\text{C}$	2.06	84.420
69	$^8\text{Li} + \text{p} \rightarrow \text{n} + \alpha + {}^4\text{He}$	3.54	177.713
70	$^8\text{B} + \text{n} \rightarrow \text{p} + \alpha + {}^4\text{He}$	3.55	218.787
71	$^9\text{Be} + \text{p} \rightarrow \text{d} + \alpha + {}^4\text{He}$	0.796	7.554
72	$^{11}\text{B} + \text{p} \rightarrow 2a + {}^4\text{He}$	3.45	100.753
73	$^{11}\text{C} + \text{n} \rightarrow 2a + {}^4\text{He}$	3.46	132.838
74	$^{12}\text{C} + \text{n} \rightarrow \gamma + {}^{13}\text{C}$	0.898	57.400
75	$^{13}\text{C} + \text{n} \rightarrow \gamma + {}^{14}\text{C}$	3.62	94.884
76	$^{14}\text{N} + \text{n} \rightarrow \gamma + {}^{15}\text{N}$	2.74	125.715
77	$^{13}\text{N} + \text{n} \rightarrow \text{p} + {}^{13}\text{C}$	1.001	34.846
78	$^{14}\text{N} + \text{n} \rightarrow \text{p} + {}^{14}\text{C}$	3.00	7.263
79	$^{15}\text{O} + \text{n} \rightarrow \text{p} + {}^{15}\text{N}$	1.001	41.037
80	$^{15}\text{O} + \text{n} \rightarrow \alpha + {}^{12}\text{C}$	0.707	98.659
81	$^{12}\text{C} + \text{p} \rightarrow \gamma + {}^{13}\text{N}$	0.896	22.554
82	$^{13}\text{C} + \text{p} \rightarrow \gamma + {}^{14}\text{N}$	1.21	87.621
83	$^{14}\text{C} + \text{p} \rightarrow \gamma + {}^{15}\text{N}$	0.912	118.452
84	$^{13}\text{N} + \text{p} \rightarrow \gamma + {}^{14}\text{O}$	3.62	53.705
85	$^{14}\text{N} + \text{p} \rightarrow \gamma + {}^{15}\text{O}$	2.73	84.678
86	$^{15}\text{N} + \text{p} \rightarrow \gamma + {}^{16}\text{O}$	3.67	140.733
87	$^{15}\text{N} + \text{p} \rightarrow \alpha + {}^{12}\text{C}$	0.706	57.622
88	$^{12}\text{C} + \alpha \rightarrow \gamma + {}^{16}\text{O}$	5.20	83.111
89	$^{10}\text{B} + \alpha \rightarrow \text{p} + {}^{13}\text{C}$	9.35	47.134
90	$^{11}\text{B} + \alpha \rightarrow \text{p} + {}^{14}\text{C}$	11.03	9.098
91	$^{11}\text{C} + \alpha \rightarrow \text{p} + {}^{14}\text{N}$	3.68	33.921
92	$^{12}\text{N} + \alpha \rightarrow \text{p} + {}^{15}\text{O}$	4.25	111.620
93	$^{13}\text{N} + \alpha \rightarrow \text{p} + {}^{16}\text{O}$	5.80	60.557
94	$^{10}\text{B} + \alpha \rightarrow \text{n} + {}^{13}\text{N}$	9.34	12.288
95	$^{11}\text{B} + \alpha \rightarrow \text{n} + {}^{14}\text{N}$	3.67	1.835
96	$^{12}\text{B} + \alpha \rightarrow \text{n} + {}^{15}\text{N}$	4.25	88.439
97	$^{13}\text{C} + \alpha \rightarrow \text{n} + {}^{16}\text{O}$	5.79	25.711
98	$^{11}\text{B} + \text{d} \rightarrow \text{p} + {}^{12}\text{B}$	4.96	13.296
99	$^{12}\text{C} + \text{d} \rightarrow \text{p} + {}^{13}\text{C}$	1.88	31.585
100	$^{13}\text{C} + \text{d} \rightarrow \text{p} + {}^{14}\text{C}$	7.58	69.069